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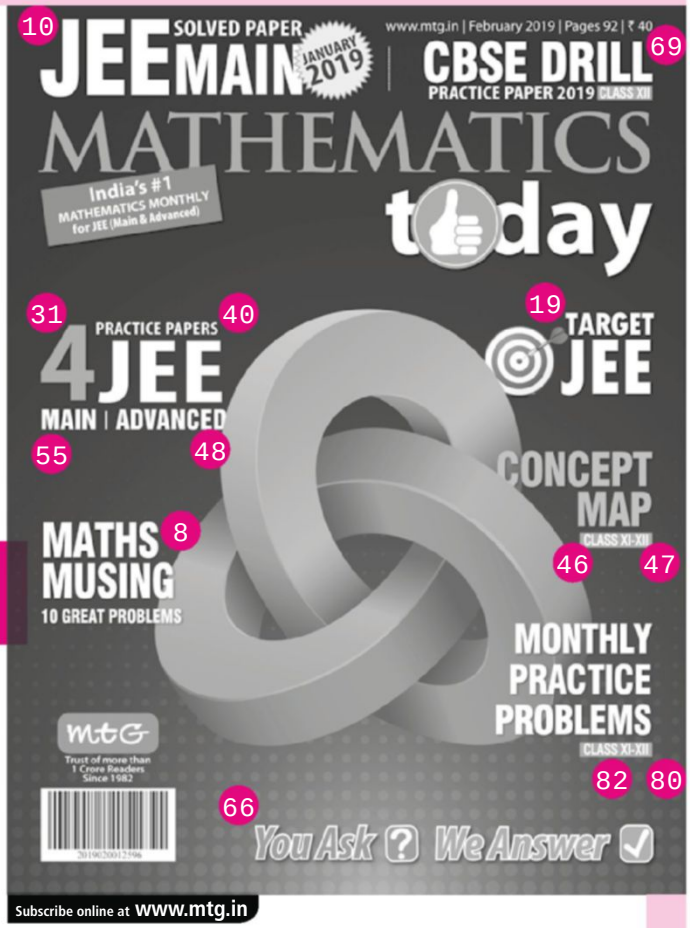

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MATHS MUSING

Maths Musing was started in January 2003 issue of Mathematics Today. The aim of Maths Musing is to augment the chances of bright students seeking admission into IITs with additional study material.
During the last 10 years there have been several changes in JEE pattern. To suit these changes Maths Musing also adopted the new pattern by changing the style of problems. Some of the Maths Musing problems have been adapted in JEE benefitting thousand of our readers. It is heartening that we receive solutions of Maths Musing problems from all over India.
Maths Musing has been receiving tremendous response from candidates preparing for JEE and teachers coaching them. We do hope that students will continue to use Maths Musing to boost up their ranks in JEE Main and Advanced.

PROBLEM Set 194

JEE MAIN

1. The cosine of obtuse angle formed by medians drawn from the vertices of the acute angles of an isosceles right angled triangle is

(a) $-\frac{1}{5}$ (b) $-\frac{2}{5}$ (c) $-\frac{3}{5}$ (d) $-\frac{4}{5}$

2. The shortest distance from the point (2, –7) to the circle $x^2 + y^2 - 14x - 10y - 151 = 0$ is

(a) 1 (b) 2 (c) 3 (d) 4

3. The difference between the greatest and the least values of the function $f(x)=\int_0^x (at^2+1+\cos t)\,dt$, $a > 0$ for $x \in [2, 3]$ is

(a) $\frac{19}{3}a+1+(\sin 3-\sin 2)$

(b) $\frac{18}{3}a+1+2\sin 3$ (c) $\frac{18}{3}a-1+2\sin 3$

(d) None of these

4. If $2x^2 + xy - 3y^2 = 0$ is the equation of pair of conjugate diameters of an ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$, then its eccentricity is

(a) $\frac{1}{\sqrt{3}}$ (b) $\frac{1}{3}$ (c) $\frac{2}{3}$ (d) $\frac{2}{\sqrt{3}}$

5. If $A=\int_0^{\pi}\frac{\cos x}{(x+2)^2}dx$, then $\int_0^{\pi/2}\frac{\sin 2x}{(x+1)}dx$ is equal to

(a) $A-\frac{1}{2}-\frac{1}{\pi+2}$ (b) $\frac{1}{2}+\frac{1}{\pi+2}-A$

(c) $\frac{1}{\pi+2}-A$ (d) $1+\frac{1}{\pi+2}-A$

JEE ADVANCED

6. If z_1, z_2 and z_3 are vertices of an equilateral triangle whose orthocentre is the origin, then

(a) $z_1 + z_2 + z_3 = 0$

(b) $|z_1| = |z_2| = |z_3|$ (c) $z_2 z_3 + z_3 z_1 + z_1 z_2 = 0$

(d) Oz_1 is perpendicular to $z_2 z_3$.

COMPREHENSION

Let $f(x)=\frac{x^2-x+1}{x^2+x+1}$ for $x \in R$, $g(x) = e^x$ for $x \in R$ and $h(x) = \tan x$ for $-\frac{\pi}{2}<x<\frac{\pi}{2}$.

7. Range of the function *foh* is

(a) $\left(\frac{1}{3}, 3\right)$ (b) $\left[\frac{1}{3}, 3\right]$

(c) $\left[\frac{1}{3}, 1\right]$ (d) [1, 3]

8. Range of the function *goh* is

(a) R (b) $[0, \infty)$

(c) $(-\infty, 0]$ (d) $(0, \infty)$

NUMERICAL ANSWER TYPE

9. The position vectors of the points A, B, C and D are $3\hat{i}-2\hat{j}-\hat{k}$, $2\hat{i}+3\hat{j}-4\hat{k}$, $-\hat{i}+\hat{j}+2\hat{k}$ and $4\hat{i}+5\hat{j}+\lambda\hat{k}$. It is known that these points are coplanar, then the value of -17λ =

MATRIX MATCH

10. A root of the equation on L.H.S satisfies R.H.S

Column-I		Column-II	
(P)	$7\cos^2 x + \sin x \cos x - 3 = 0$	(1)	$\tan x = 4/3$
(Q)	$2\sin^2((\pi/2)\cos^2 x) = 1 - \cos(\pi \sin 2x)$	(2)	$\tan x = -1$
(R)	$6\sec^2 x - 11\tan x - 2 = 0$	(3)	$\tan x = 1/2$
(S)	$2\cos 2x - \sin 2x = 2\sin^2 x$	(4)	$\cos 2x = 3/5$

	P	Q	R	S
(a)	2, 3	3, 4	1, 2	2, 4
(b)	1, 2	3, 4	1, 3, 4	2, 3, 4
(c)	1, 2	3, 4	2, 3	1, 3, 4
(d)	2, 3	3, 4	1, 3, 4	2, 4

See Solution Set of Maths Musing 193 on page no. 85

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$$m(4m^2 + m + 12) = 3(p^n - 1).$$

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JEE MAIN

Held on 12th January (Morning Shift)

SOLVED PAPER 2019

1. The sum of the distinct real values of μ, for which the vectors, $\mu\hat{i}+\hat{j}+\hat{k}$, $\hat{i}+\mu\hat{j}+\hat{k}$, $\hat{i}+\hat{j}+\mu\hat{k}$ are co-planar, is

(a) 2 (b) 0 (c) –1 (d) 1

2. The area (in sq. units) of the region bounded by the parabola, $y = x^2 + 2$ and the lines, $y = x + 1$, $x = 0$ and $x = 3$, is

(a) 15/2 (b) 17/4 (c) 21/2 (d) 15/4

3. Let C_1 and C_2 be the centres of the circles $x^2 + y^2 - 2x - 2y - 2 = 0$ and $x^2 + y^2 - 6x - 6y + 14 = 0$ respectively. If P and Q are the points of intersection of these circles, then the area (in sq. units) of the quadrilateral PC_1QC_2 is

(a) 8 (b) 4 (c) 6 (d) 9

4. Consider three boxes, each containing 10 balls labelled 1, 2, ..., 10. Suppose one ball is randomly drawn from each of the boxes. Denote by n_i, the label of the ball drawn from the i^{th} box, (i = 1, 2, 3). Then, the number of ways in which the balls can be chosen such that $n_1 < n_2 < n_3$ is

(a) 120 (b) 164 (c) 240 (d) 82

5. Considering only the principal values of inverse functions, the set

$$A=\left\{x\geq 0:\tan^{-1}(2x)+\tan^{-1}(3x)=\frac{\pi}{4}\right\}$$

(a) contains two elements

(b) contains more than two elements

(c) is an empty set (d) is a singleton

6. The perpendicular distance from the origin to the plane containing the two lines, $\frac{x+2}{3}=\frac{y-2}{5}=\frac{z+5}{7}$ and $\frac{x-1}{1}=\frac{y-4}{4}=\frac{z+4}{7}$, is

(a) 11 (b) $6\sqrt{11}$ (c) $\frac{11}{\sqrt{6}}$ (d) $11\sqrt{6}$

7. If $\frac{z-\alpha}{z+\alpha}(\alpha\in R)$ is a purely imaginary number and $|z| = 2$, then a value of α is

(a) 1 (b) $\sqrt{2}$ (c) $\frac{1}{2}$ (d) 2

8. If the straight line, $2x - 3y + 17 = 0$ is perpendicular to the line passing through the points (7, 17) and (15, β), then β equals

(a) 5 (b) $\frac{35}{3}$ (c) $-\frac{35}{3}$ (d) –5

9. The boolean expression $((p \wedge q) \vee (p \vee \sim q)) \wedge (\sim p \wedge \sim q)$ is equivalent to

(a) $(\sim p) \wedge (\sim q)$ (b) $p \wedge (\sim q)$

(c) $p \wedge q$ (d) $p \vee (\sim q)$

10. Let $S_k=\frac{1+2+3+...+k}{k}$. If $S_1^2+S_2^2+...+S_{10}^2=\frac{5}{12}A$, then A is equal to

(a) 303 (b) 283 (c) 301 (d) 156

11. An ordered pair(α, β) for which the system of linear equations

$(1 + \alpha)x + \beta y + z = 2$, $\alpha x + (1 + \beta)y + z = 3$,

$\alpha x + \beta y + 2z = 2$ has a unique solution, is

(a) (–3, 1) (b) (2, 4) (c) (1, –3) (d) (–4, 2)

12. The maximum area (in sq. units) of a rectangle having its base on the x-axis and its other two vertices on the parabola, $y = 12 - x^2$ such that the rectangle lies inside the parabola, is

(a) 36 (b) $18\sqrt{3}$ (c) $20\sqrt{2}$ (d) 32

13. Let $S = \{1, 2, 3, ..., 100\}$. The number of non-empty subsets A of S such that the product of elements in A is even is

(a) $2^{50}(2^{50} - 1)$ (b) $2^{50} - 1$

(c) $2^{50} + 1$ (d) $2^{100} - 1$

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14. Let f and g be continuous functions on $[0, a]$ such that $f(x) = f(a - x)$ and $g(x) + g(a - x) = 4$, then $\int_0^a f(x)g(x)\,dx$ is equal to

(a) $2\int_0^a f(x)dx$ (b) $4\int_0^a f(x)dx$

(c) $-3\int_0^a f(x)dx$ (d) $\int_0^a f(x)dx$

15. If $x > 1$ for $(2x)^{2y} = 4e^{2x-2y}$, then $(1+\log_e 2x)^2 \frac{dy}{dx}$ equals

(a) $\frac{x\log_e 2x - \log_e 2}{x}$ (b) $\log_e 2x$

(c) $\frac{x\log_e 2x + \log_e 2}{x}$ (d) $x\log_e 2x$

16. If a variable line, $3x + 4y - \lambda = 0$ is such that the two circles $x^2 + y^2 - 2x - 2y + 1 = 0$ and $x^2 + y^2 - 18x - 2y + 78 = 0$ are on its opposite sides, then the set of all values of λ is the interval

(a) (23, 31) (b) (2, 17) (c) [13, 23] (d) [12, 21]

17. If λ be the ratio of the roots of the quadratic equation in x, $3m^2x^2 + m(m - 4)x + 2 = 0$, then the least value of m for which $\lambda + \frac{1}{\lambda} = 1$, is

(a) $-2+\sqrt{2}$ (b) $4-3\sqrt{2}$

(c) $4-2\sqrt{3}$ (d) $2-\sqrt{3}$

18. In a random experiment, a fair die is rolled until two fours are obtained in succession. The probability that the experiment will end in the fifth throw of the die is equal to

(a) $\frac{150}{6^5}$ (b) $\frac{225}{6^5}$ (c) $\frac{175}{6^5}$ (d) $\frac{200}{6^5}$

19. The maximum value of the expression $3\cos\theta + 5\sin\left(\theta - \frac{\pi}{6}\right)$ for any real value of θ is

(a) $\frac{\sqrt{79}}{2}$ (b) $\sqrt{31}$ (c) $\sqrt{34}$ (d) $\sqrt{19}$

20. If the sum of the deviations of 50 observations from 30 is 50, then the mean of these observations is

(a) 30 (b) 51 (c) 31 (d) 50

21. Let $y = y(x)$ be the solution of the differential equation, $x\frac{dy}{dx} + y = x\log_e x$, $(x > 1)$. If $2y(2) = \log_e 4 - 1$, then $y(e)$ is equal to

(a) $\frac{e^2}{4}$ (b) $-\frac{e^2}{2}$ (c) $-\frac{e}{2}$ (d) $\frac{e}{4}$

22. If the vertices of a hyperbola be at $(-2, 0)$ and $(2, 0)$ and one of its foci be at $(-3, 0)$, then which one of the following points does not lie on this hyperbola?

(a) $(2\sqrt{6}, 5)$ (b) $(-6, 2\sqrt{10})$

(c) $(6, 5\sqrt{2})$ (d) $(4, \sqrt{15})$

23. Let $P = \begin{bmatrix} 1 & 0 & 0 \\ 3 & 1 & 0 \\ 9 & 3 & 1 \end{bmatrix}$ and $Q = [q_{ij}]$ be two 3×3 matrices such that $Q - P^5 = I_3$. Then $\frac{q_{21} + q_{31}}{q_{32}}$ is equal to

(a) 10 (b) 135 (c) 9 (d) 15

24. Integral $\int \cos(\log_e x)dx$ equals (where C is the constant of integration)

(a) $\frac{x}{2}[\sin(\log_e x) - \cos(\log_e x)] + C$

(b) $x[\cos(\log_e x) - \sin(\log_e x)] + C$

(c) $\frac{x}{2}[\cos(\log_e x) + \sin(\log_e x)] + C$

(d) $x[\cos(\log_e x) + \sin(\log_e x)] + C$

25. Let $P(4, -4)$ and $Q(9, 6)$ be two points on the parabola, $y^2 = 4x$ and let X be any point on the arc POQ of this parabola, where O is the vertex of this parabola, such that the area of ΔPXQ is maximum. Then this maximum area (in sq. units) is

(a) $\frac{125}{2}$ (b) $\frac{125}{4}$ (c) $\frac{625}{4}$ (d) $\frac{75}{2}$

26. The product of three consecutive terms of a G.P. is 512. If 4 is added to each of the first and the second of these terms, the three terms now form an A.P. Then the sum of the original three terms of the given G.P. is

(a) 36 (b) 24 (c) 28 (d) 32

27. $\lim_{x\to\pi/4} \frac{\cot^3 x - \tan x}{\cos\left(x + \frac{\pi}{4}\right)}$ is

(a) $4\sqrt{2}$ (b) $8\sqrt{2}$ (c) 4 (d) 8

28. A ratio of the 5^{th} term from the beginning to the 5^{th} term from the end in the binomial expansion of $\left(2^{1/3} + \frac{1}{2(3)^{1/3}}\right)^{10}$ is

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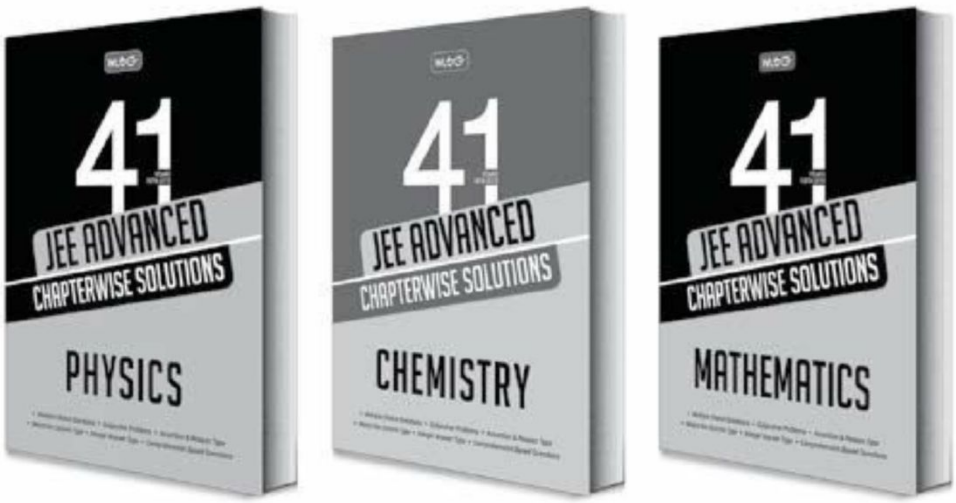

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(a) $4(36)^{1/3}:1$ (b) $1:2(6)^{1/3}$

(c) $2(36)^{1/3}:1$ (d) $1:4(16)^{1/3}$

29. Let S be the set of all points in $(-\pi, \pi)$ at which the function, $f(x) = \min\{\sin x, \cos x\}$ is not differentiable. Then S is a subset of which of the following?

(a) $\left\{-\frac{\pi}{4}, 0, \frac{\pi}{4}\right\}$ (b) $\left\{-\frac{3\pi}{4}, -\frac{\pi}{4}, \frac{3\pi}{4}, \frac{\pi}{4}\right\}$

(c) $\left\{-\frac{3\pi}{4}, -\frac{\pi}{2}, \frac{\pi}{2}, \frac{3\pi}{4}\right\}$ (d) $\left\{-\frac{\pi}{2}, -\frac{\pi}{4}, \frac{\pi}{4}, \frac{\pi}{2}\right\}$

30. A tetrahedron has vertices $P(1, 2, 1)$, $Q(2, 1, 3)$, $R(-1, 1, 2)$ and $O(0, 0, 0)$. The angle between the faces OPQ and PQR is

(a) $\cos^{-1}\left(\frac{7}{31}\right)$ (b) $\cos^{-1}\left(\frac{17}{31}\right)$

(c) $\cos^{-1}\left(\frac{19}{35}\right)$ (d) $\cos^{-1}\left(\frac{9}{35}\right)$

SOLUTIONS

1. (c) : Let $\vec{a} = \mu\hat{i} + \hat{j} + \hat{k}, \vec{b} = \hat{i} + \mu\hat{j} + \hat{k}$ and $\vec{c} = \hat{i} + \hat{j} + \mu\hat{k}$

$$\therefore [\vec{a}\ \vec{b}\ \vec{c}] = \begin{vmatrix} \mu & 1 & 1 \\ 1 & \mu & 1 \\ 1 & 1 & \mu \end{vmatrix}$$

$= \mu(\mu^2 - 1) - 1(\mu - 1) + 1(1 - \mu)$

$= \mu^3 - \mu - \mu + 1 + 1 - \mu = \mu^3 - 3\mu + 2$

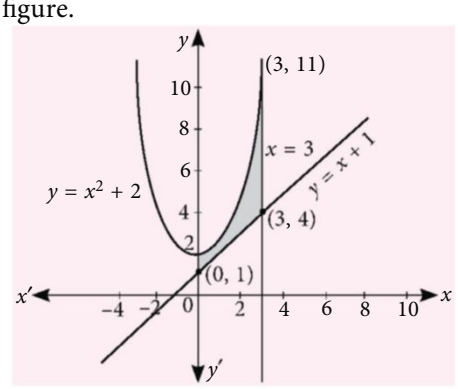
$\vec{a}, \vec{b}$ and $\vec{c}$ are coplanar, so $[\vec{a}\ \vec{b}\ \vec{c}] = 0$

$\therefore$ $\mu^3 - 3\mu + 2 = 0 \Rightarrow (\mu - 1)^2(\mu + 2) = 0$

$\Rightarrow$ $\mu = 1, 1, -2$

$\therefore$ The sum of the distinct real values of μ $= 1 - 2 = -1$.

2. (a) : The shaded region represents the required area in the figure.



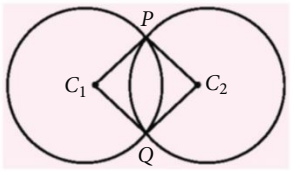
$$\text{Required area} = \int_0^3 [(x^2 + 2) - (x + 1)]\,dx = \int_0^3 (x^2 - x + 1)\,dx$$

$$= \left[\frac{x^3}{3} - \frac{x^2}{2} + x\right]_0^3 = 9 - \frac{9}{2} + 3 = \frac{15}{2} \text{ sq. units}$$

3. (b) : Let $S_1 \equiv x^2 + y^2 - 2x - 2y - 2 = 0$

$S_2 \equiv x^2 + y^2 - 6x - 6y + 14 = 0$

$\therefore$ $C_1 \equiv (1, 1)$ and $C_2 \equiv (3, 3)$.



Radius of S_1, $PC_1 = \sqrt{1+1+2} = 2$

Radius of S_2, $PC_2 = \sqrt{9+9-14} = 2$

$\therefore$ $PC_1 = QC_1 = PC_2 = QC_2 = 2$

Now, $2g_1g_2 + 2f_1f_2$

$= 2 \times 3 + 2 \times 3 = 6 + 6 = 12$ and $c_1 + c_2 = 14 - 2 = 12$

Here, $2g_1g_2 + 2f_1f_2 = c_1 + c_2$

$\therefore$ Both circles are orthogonal. So, PC_1QC_2 is a square.

$\therefore$ Area of $PC_1QC_2 = 2 \times 2 = 4$ sq. units

4. (a) : Let $n_1 = 1$, then n_2 can be 2, 3, ..., 9 and n_3 can be 3, ..., 10

$\therefore$ No. of ways $= 8 + 7 + 6 + 5 + 4 + 3 + 2 + 1 = \frac{8 \times 9}{2}$

Similarly, when $n_1 = 2$, then n_2 can be 3, ..., 9 and n_3 can be 4, ..., 10

$\therefore$ No. of ways $= 7 + 6 + 5 + 4 + 3 + 2 + 1 = \frac{7 \times 8}{2}$

And so on.

$$\therefore \text{Total required ways} = \frac{8 \times 9}{2} + \frac{7 \times 8}{2} + ... + \frac{2 \times 3}{2} + \frac{1 \times 2}{2}$$

$$= \frac{240}{2} = 120$$

5. (d) : Here, $A = \left\{x \geq 0 : \tan^{-1}(2x) + \tan^{-1}(3x) = \frac{\pi}{4}\right\}$

Now, $\tan^{-1}(2x) + \tan^{-1}(3x) = \frac{\pi}{4}$

$$\Rightarrow \tan^{-1}\left(\frac{2x + 3x}{1 - 2x \times 3x}\right) = \frac{\pi}{4} \Rightarrow \tan^{-1}\left(\frac{5x}{1 - 6x^2}\right) = \frac{\pi}{4}$$

$$\Rightarrow \frac{5x}{1 - 6x^2} = \tan\frac{\pi}{4} = 1 \Rightarrow 5x = 1 - 6x^2$$

$\Rightarrow$ $6x^2 + 5x - 1 = 0 \Rightarrow (6x - 1)(x + 1) = 0$

$$\Rightarrow x = \frac{1}{6} \qquad [\because x \geq 0]$$

6. (c) : Let $L_1 : \frac{x+2}{3} = \frac{y-2}{5} = \frac{z+5}{7}$ and

$L_2 : \frac{x-1}{1} = \frac{y-4}{4} = \frac{z+4}{7}$

$\therefore$ Equation of plane containing L_1 and L_2 is

$$\begin{vmatrix} x-1 & y-4 & z+4 \\ 3 & 5 & 7 \\ 1 & 4 & 7 \end{vmatrix} = 0$$

$\Rightarrow$ $(x-1)(35-28)-(y-4)(21-7)+(z+4)(12-5)=0$

$\Rightarrow$ $7x-14y+7z+77=0 \Rightarrow x-2y+z+11=0$

$\therefore$ Perpendicular distance from the origin to the plane is

$\frac{|11|}{\sqrt{1+4+1}} = \frac{11}{\sqrt{6}}$ units

7. (d) : Let $z = x + iy$

$\therefore$ $|z| = \sqrt{x^2+y^2} = 2 \Rightarrow x^2+y^2 = 4$

Now, $\frac{z-\alpha}{z+\alpha} = \frac{x+iy-\alpha}{x+iy+\alpha} = \frac{(x-\alpha)+iy}{(x+\alpha)+iy} \times \frac{(x+\alpha)-iy}{(x+\alpha)-iy}$

$= \frac{(x^2+y^2-\alpha^2)}{(x+\alpha)^2+y^2} + \frac{i2\alpha y}{(x+\alpha)^2+y^2}$

$\Rightarrow$ $\frac{x^2+y^2-\alpha^2}{(x+\alpha)^2+y^2} = 0$ $\left[\because \frac{z-\alpha}{z+\alpha} \text{ is purely imaginary}\right]$

$\Rightarrow$ $x^2+y^2-\alpha^2=0 \Rightarrow \alpha^2=4 \Rightarrow \alpha = \pm 2$

8. (a) : Let m_1 = slope of line $2x - 3y + 17 = 0$ and m_2 = slope of line joining (7, 17) and (15, β)

$\therefore$ $m_1 = \frac{2}{3}$ and $m_2 = \frac{\beta-17}{15-7} = \frac{\beta-17}{8}$

Since, both the lines are perpendicular.

$\therefore$ $m_1 m_2 = -1$

$\Rightarrow$ $\frac{2}{3} \times \frac{\beta-17}{8} = -1 \Rightarrow \beta - 17 = -12 \Rightarrow \beta = 5$

9. (a) : $((p \wedge q) \vee (p \vee \sim q)) \wedge (\sim p \wedge \sim q)$

$\equiv ((p \vee (p \vee \sim q)) \wedge (q \vee (p \vee \sim q))) \wedge (\sim p \wedge \sim q)$

$\equiv ((p \vee \sim q) \wedge (q \vee \sim q \vee p)) \wedge (\sim p \wedge \sim q)$

$\equiv ((p \vee \sim q) \wedge (t \vee p)) \wedge (\sim p \wedge \sim q)$

$\equiv ((p \vee \sim q) \wedge t) \wedge (\sim p \wedge \sim q)$

$\equiv (p \vee \sim q) \wedge (\sim p \wedge \sim q)$

$\equiv (p \wedge \sim p \wedge \sim q) \vee (\sim q \wedge \sim p \wedge \sim q)$

$\equiv (c \wedge \sim q) \vee (\sim q \wedge \sim p)$

$\equiv c \vee (\sim q \wedge \sim p)$

$\equiv \sim p \wedge \sim q$

10. (a) : $S_k = \frac{k(k+1)}{2k} = \frac{k+1}{2}$

Now, $\sum_{k=1}^{10} (S_k)^2 = \frac{5}{12}A \Rightarrow \sum_{k=1}^{10} \left(\frac{k+1}{2}\right)^2 = \frac{5}{12}A$

$\Rightarrow$ $\frac{1}{4}(2^2+3^2+....+11^2) = \frac{5}{12}A$

$\Rightarrow$ $\frac{1}{4}\left(\frac{11\times12\times23}{6} - 1\right) = \frac{5}{12}A$

$\Rightarrow$ $\frac{505}{4} = \frac{5}{12}A \Rightarrow A = 303$

11. (b) : For unique solution

$$\begin{vmatrix} 1+\alpha & \beta & 1 \\ \alpha & 1+\beta & 1 \\ \alpha & \beta & 2 \end{vmatrix} \neq 0$$

$\Rightarrow (1+\alpha)(2+2\beta-\beta) - \beta(2\alpha-\alpha) + 1(\alpha\beta-\alpha-\alpha\beta) \neq 0$

$\Rightarrow$ $2+2\beta-\beta+2\alpha+2\alpha\beta-\alpha\beta-2\alpha\beta+\alpha\beta-\alpha \neq 0$

$\Rightarrow$ $2+\alpha+\beta \neq 0$

$\therefore$ (2, 4) satisfies the above condition.

12. (d) : Let $(a, 0)$ be any point on the x-axis, which is the vertex of the rectangle

So, the co-ordinates of the vertex of the rectangle lying on the parabola $y = 12 - x^2$ is $(a, 12-a^2)$.

$\therefore$ Area of rectangle, $f(a) = 2a(12 - a^2)$

$\therefore$ $f'(a) = 2(12 - 3a^2)$

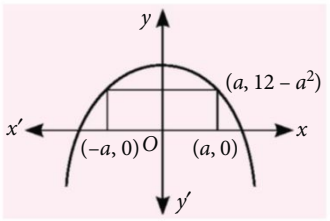
For maximum area

$f'(a) = 0$

$\Rightarrow$ $2(12 - 3a^2) = 0$

$\Rightarrow$ $a = \pm 2$

$\therefore$ Maximum area at $a = 2$ is $f(2) = 32$ sq. units



13. (a) : Here, $S = \{1, 2, 3, ..., 100\}$

So, the number of non-empty subsets A of S such that the product of elements in A is even

= Total non empty subsets–subsets with product of element is odd

$= 2^{100} - 1 - [2^{50} - 1] = 2^{100} - 2^{50} = 2^{50}(2^{50} - 1)$

14. (a) : Here, $f(x) = f(a - x)$ and $g(x) + g(a - x) = 4$

Let $I = \int_0^a f(x)g(x)\,dx = \int_0^a f(a-x)g(a-x)\,dx$

$= \int_0^a f(x)(4-g(x))\,dx = 4\int_0^a f(x)\,dx - \int_0^a f(x)g(x)\,dx$

$= 4\int_0^a f(x)\,dx - I \Rightarrow I = 2\int_0^a f(x)\,dx$

MPP-10 CLASS XII ANSWER KEY

1.	(c)	**2.**	(c)	**3.**	(b)	**4.**	(c)	**5.**	(c)
6.	(d)	**7.**	(b)	**8.**	(a,b,c,d)			**9.**	(a,b,c)
10.	(a,d)	**11.**	(c,d)	**12.**	(a,c,d)	**13.**	(b)	**14.**	(c)
15.	(d)	**16.**	(b)	**17.**	(6)	**18.**	(3)	**19.**	(1)
20.	(1)								

15. (a) : $(2x)^{2y} = 4e^{2x-2y}$

$\Rightarrow \quad 2y \log_e 2x = \log_e 4 + 2x - 2y$

$\Rightarrow \quad 2y \log_e 2x = 2\log_e 2 + 2x - 2y$

$\Rightarrow \quad y \log_e 2x = \log_e 2 + x - y \quad \Rightarrow \quad y = \dfrac{x + \log_e 2}{1 + \log_e 2x}$

$$\therefore \quad \frac{dy}{dx} = \frac{1 + \log_e 2x - (x + \log_e 2)\frac{1}{x}}{(1 + \log_e 2x)^2}$$

$$\Rightarrow \quad \frac{dy}{dx}(1 + \log_e 2x)^2 = \frac{x \log_e 2x - \log_e 2}{x}$$

16. (d) : Let $S_1 : x^2 + y^2 - 2x - 2y + 1 = 0$

$C_1(1, 1), r_1 = 1$

and $S_2 : x^2 + y^2 - 18x - 2y + 78 = 0$

$C_2 (9, 1), r_2 = 2$

Centre of circles lie on opposite side of line

$3x + 4y - \lambda = 0$

$\therefore \quad (3 + 4 - \lambda)(27 + 4 - \lambda) < 0$

$\Rightarrow \quad (\lambda - 7)(\lambda - 31) < 0 \quad \Rightarrow \quad \lambda \in (7, 31)$

Line lies outside the circles S_1 and S_2.

$$\therefore \quad \left|\frac{3+4-\lambda}{5}\right| \geq 1 \Rightarrow \lambda \in (-\infty, 2] \cup [12, \infty)$$

and $$\left|\frac{27+4-\lambda}{5}\right| \geq 2 \Rightarrow \lambda \in (-\infty, 21] \cup [41, \infty)$$

So, $\lambda \in [12, 21]$

17. (b) : Here, $3m^2x^2 + m(m - 4)x + 2 = 0$

Let α, β be the roots of the given equation

$$\therefore \quad \lambda = \frac{\alpha}{\beta}, \alpha + \beta = \frac{m(4-m)}{3m^2} = \frac{4-m}{3m}, \alpha\beta = \frac{2}{3m^2}$$

Given, $\lambda + \dfrac{1}{\lambda} = 1$

$$\Rightarrow \quad \frac{\alpha}{\beta} + \frac{\beta}{\alpha} = 1 \Rightarrow \frac{\alpha^2 + \beta^2}{\alpha\beta} = 1 \Rightarrow \alpha^2 + \beta^2 = \alpha\beta$$

Now, $(\alpha + \beta)^2 = \alpha^2 + \beta^2 + 2\alpha\beta = 3\alpha\beta$

$$\Rightarrow \quad \frac{(4-m)^2}{9m^2} = \frac{2}{m^2} \quad \Rightarrow \quad (4 - m)^2 = 18$$

$$\Rightarrow \quad 4 - m = \pm 3\sqrt{2} \quad \Rightarrow \quad m = 4 \pm 3\sqrt{2}$$

$\therefore \quad$ Least value of $m = 4 - 3\sqrt{2}$

18. (c) : Let p be the success that die turns up 4.

$\therefore \quad p = \dfrac{1}{6}, q = \dfrac{5}{6}$. Here $n = 5$

$$\therefore \quad \text{Required probability} = \left(\frac{5}{6}\right)^3\left(\frac{1}{6}\right)^2 + {}^2C_1\left(\frac{5}{6}\right)^2\left(\frac{1}{6}\right)^3$$

$$= \frac{1}{6^5}(5^3 + 2 \times 5^2) = \frac{175}{6^5}$$

19. (d) : Let $x = 3\cos\theta + 5\sin\left(\theta - \dfrac{\pi}{6}\right)$

$$= 3\cos\theta + 5\left(\frac{\sqrt{3}}{2}\sin\theta - \frac{1}{2}\cos\theta\right) = \frac{5\sqrt{3}}{2}\sin\theta + \frac{1}{2}\cos\theta$$

So, maximum value of x for real θ is $\sqrt{\dfrac{75}{4} + \dfrac{1}{4}} = \sqrt{19}$

20. (c) : Here, $\displaystyle\sum_{i=1}^{50}(x_i - 30) = 50$

$$\Rightarrow \quad \sum_{i=1}^{50} x_i = 50 + 50 \times 30$$

$$\therefore \quad \text{Mean} = \frac{\displaystyle\sum_{i=1}^{50} x_i}{50} = \frac{50 \times 31}{50} = 31$$

21. (d) : Here, $\dfrac{dy}{dx} + \dfrac{y}{x} = \log_e x$

$$\therefore \quad \text{I.F.} = e^{\int \frac{1}{x} dx} = x$$

The required solution is $xy = \int \log_e x \cdot x dx + C$

$$\Rightarrow \quad xy = \frac{x^2}{2}\log_e x - \frac{x^2}{4} + C \qquad \ldots(i)$$

$\because \quad 2y(2) = \log_e 4 - 1 \Rightarrow 2y(2) = 2\log_e 2 - 1$

From (i), $C = 0$

So, $y = \dfrac{x}{2}\log_e x - \dfrac{x}{4} \quad \Rightarrow \quad y(e) = \dfrac{e}{2} - \dfrac{e}{4} = \dfrac{e}{4}$

22. (c) : Here $a = 2$, $ae = 3 \Rightarrow e = \dfrac{3}{2}$

$\therefore \quad b^2 = a^2e^2 - a^2 = 9 - 4 = 5$

So, the equation of hyperbola is

$$\frac{x^2}{4} - \frac{y^2}{5} = 1$$

The point $(6, 5\sqrt{2})$ does not lie on this hyperbola.

23. (a) : Here, $P = \begin{bmatrix} 1 & 0 & 0 \\ 3 & 1 & 0 \\ 9 & 3 & 1 \end{bmatrix}$ and $Q = P^5 + I_3$

$$\therefore \quad P^2 = \begin{bmatrix} 1 & 0 & 0 \\ 3 & 1 & 0 \\ 9 & 3 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 & 0 \\ 3 & 1 & 0 \\ 9 & 3 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 6 & 1 & 0 \\ 27 & 6 & 1 \end{bmatrix}$$

$$P^3 = \begin{bmatrix} 1 & 0 & 0 \\ 6 & 1 & 0 \\ 27 & 6 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 & 0 \\ 3 & 1 & 0 \\ 9 & 3 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 9 & 1 & 0 \\ 54 & 9 & 1 \end{bmatrix}$$

Similarly, $P^5 = \begin{bmatrix} 1 & 0 & 0 \\ 15 & 1 & 0 \\ 135 & 15 & 1 \end{bmatrix}$

$\therefore \quad Q = \begin{bmatrix} 2 & 0 & 0 \\ 15 & 2 & 0 \\ 135 & 15 & 2 \end{bmatrix} \Rightarrow \frac{q_{21}+q_{31}}{q_{32}} = \frac{15+135}{15} = 10$

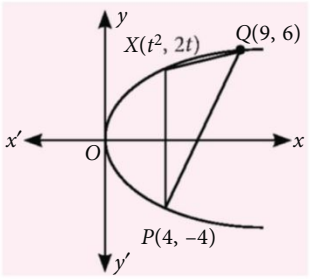
24. (c) : Let $I = \int \cos(\log_e x)\,dx$

$\Rightarrow \quad I = \cos(\log_e x)\,x + \int \sin(\log_e x)\,dx$

$\Rightarrow \quad I = x\cos(\log_e x) + \sin(\log_e x)\,x - \int \cos(\log_e x)\,dx$

$\Rightarrow \quad I = \frac{x}{2}[\cos(\log_e x) + \sin(\log_e x)] + C$

25. (b) : Let $X(t^2, 2t)$ be any point on the parabola $y^2 = 4x$.



$\therefore \quad 2yy' = 4$

$\Rightarrow \quad y' = \frac{2}{y}$

$\Rightarrow \quad (y')_{(t^2, 2t)} = \frac{1}{t} = 2$

[Since, for maximum area X should lie on tangent parallel to PQ]

$\therefore \quad t = \frac{1}{2}$

$\therefore$ Maximum area $= \left| \frac{1}{2} \begin{vmatrix} \frac{1}{4} & 1 & 1 \\ 9 & 6 & 1 \\ 4 & -4 & 1 \end{vmatrix} \right|$

$= \left| \frac{1}{2}\left[\frac{1}{4}(10) - 1(5) + 1(-60)\right] \right|$

$= \left| \frac{1}{2}\left[\frac{5}{2} - 65\right] \right| = \frac{125}{4}$ sq. units

26. (c) : Let 3 consecutive terms are $\frac{a}{r}$, a, ar of a G.P.

$\therefore \quad a^3 = 512 \Rightarrow a = 8$

Now, $\frac{8}{r} + 4$, 12, $8r$ are in A.P.

$\therefore \quad 24 = \frac{8}{r} + 4 + 8r \Rightarrow 24r = 8 + 4r + 8r^2$

$\Rightarrow \quad 2r^2 - 5r + 2 = 0 \Rightarrow (r - 2)(2r - 1) = 0$

$\Rightarrow \quad r = 2$ or $\frac{1}{2}$

When $r = 2$, terms are 4, 8, 16

When $r = \frac{1}{2}$, terms are 16, 8, 4

So, required sum = 16 + 8 + 4 = 28

27. (d) : $\lim\limits_{x \to \frac{\pi}{4}} \frac{\cot^3 x - \tan x}{\cos\left(x + \frac{\pi}{4}\right)}$

$= \lim\limits_{x \to \frac{\pi}{4}} \frac{1 - \tan^4 x}{\cos\left(x + \frac{\pi}{4}\right)} = 2 \lim\limits_{x \to \frac{\pi}{4}} \frac{1 - \tan^2 x}{\cos\left(x + \frac{\pi}{4}\right)}$

$= 2 \lim\limits_{x \to \frac{\pi}{4}} \frac{\cos^2 x - \sin^2 x}{\frac{1}{\sqrt{2}}(\cos x - \sin x)} \cdot \frac{1}{\cos^2 x}$

$= 4\sqrt{2} \lim\limits_{x \to \frac{\pi}{4}} (\cos x + \sin x) = 8$

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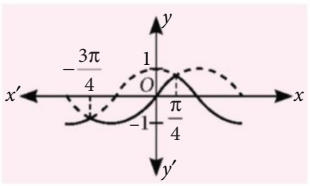
28. (a) : In the expansion $\left(2^{1/3}+\frac{1}{2(3)^{1/3}}\right)^{10}$ the ratio of 5^{th} term from the beginning to the 5^{th} term from the end

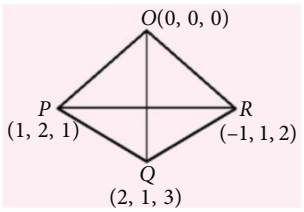
$$=\frac{{}^{10}C_4(2^{1/3})^{10-4}\left(\frac{1}{2(3)^{1/3}}\right)^4}{{}^{10}C_4\left(\frac{1}{2(3)^{1/3}}\right)^{10-4}(2^{1/3})^4}=\frac{4\cdot(36)^{1/3}}{1}$$

29. (b) : Here, $y = \min\{\sin x, \cos x\}$ is not differentiable at $\left\{-\frac{3\pi}{4}, \frac{\pi}{4}\right\}$.



30. (c) : Here, $\overrightarrow{OP}\times\overrightarrow{OQ}=(\hat{i}+2\hat{j}+\hat{k})\times(2\hat{i}+\hat{j}+3\hat{k})$

$$=5\hat{i}-\hat{j}-3\hat{k}$$



Again, $\overrightarrow{PQ}\times\overrightarrow{PR}=(\hat{i}-\hat{j}+2\hat{k})\times(-2\hat{i}-\hat{j}+\hat{k})$

$$=\hat{i}-5\hat{j}-3\hat{k}$$

Let angle between faces OPQ and PQR is θ

$$\therefore\ \cos\theta=\frac{5+5+9}{(\sqrt{25+9+1})^2}=\frac{19}{35}.$$

Introduction of two levels of Mathematics for All India Secondary School Examination from the Academic Session ending March, 2020 onwards

NCF 2005, Position Paper by National Focus Group on Examination Reforms states that – just as we allow students and schools, some element of choice in the choosing of their subjects, they should have the choice of picking one of two levels within that subject. As per NCF, not only would the two levels of examinations cater for different kinds of learners and allow different levels of testing, it would also reduce overall student stress levels. It is well known that students experience greatest stress before and during their most 'difficult' subject exam. Keeping in view of this important aspect and as evidenced by the Board results, the Board has decided to introduce two levels of examination in Mathematics for the students who are going to appear in the Board examination for the academic session ending March 2020 onwards.

The details of this scheme are as under:

- The two levels of Examination will be held in the subject of Mathematics in the Board examination for Class X in the year 2020 and the same shall not be applicable to the internal assessment in class X.
- There shall not be two levels of Assessment/Examination for class IX.
- First level would be the same as the existing one, and the other would be an easier level.
- The nomenclature for the two Examinations will be; Mathematics-Standard for the existing level of examination, and Mathematics-Basic for the easier level of examination.
- The syllabus, class room teaching and internal assessment for both the levels of examination would remain the same; so that the students get an opportunity to study the whole range of topics throughout the year and are able to decide upon the level of Board examination depending upon their aptitude and abilities.
- The Standard level will be meant for students who wish to opt for Mathematics at Sr. Secondary level and the Basic level would be for students not keen to pursue Mathematics at higher levels.
- A student will have the right to choose between the two levels of Examination at the time of submission of List of Candidates (LoC) by the affiliated school to the Board online.
- In case student fails at any level of Mathematics, he/she can appear at the compartment examination as per norms of the Board according to the options given below :
- In case of failure in Mathematics -Basic Compartment option is Mathematics-Basic and in case of failure in Mathematics-Standard Compartment options is Mathematics Standard or Mathematics- Basic
- A student who qualifies the Mathematics-Basic, shall be given an option to appear in Mathematics-Standard at the time of Compartment exams as per norms of the Board, in case he/she changes his/her mind to pursue Mathematics at Senior Secondary level.

Design for both the levels of question papers as per curriculum for the academic session ending March 2020 onwards will be made available on the official website of CBSE, *www.cbseacademic.nic.in* in due course of time.

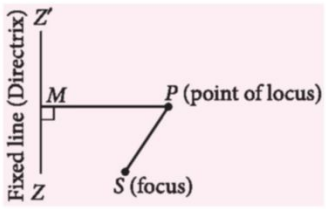
TARGET JEE

Conic Sections

DEFINITION OF CONIC

A conic is the locus of a point (P) which moves in such a way that its distance from a fixed point (S) always bears a constant ratio to its distance from a fixed straight line. The fixed point S is called focus and the fixed straight line is called directrix of the conic. The constant ratio is known as eccentricity which is denoted by e.

$\therefore \quad e = \dfrac{SP}{PM}$ ...(i)



- If $e = 0$, the conic is a circle.
- If $e = 1$, the conic is a parabola.
- If $0 < e < 1$, the conic is an ellipse.
- If $e > 1$, the conic is a hyperbola.

Let $P(x, y)$ be any point, $S(\alpha, \beta)$ be fixed point and $Ax + By + C = 0$ be the straight line, then by (i) we have, $e^2(PM)^2 = (SP)^2$

$$\Rightarrow \quad (x-\alpha)^2 + (y-\beta)^2 = \frac{e|Ax+By+C|}{\sqrt{A^2+B^2}}$$

which can be written as $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$ which is a general equation of second degree. The general equation of second degree will represent

(1) a pair of straight lines if $\begin{vmatrix} a & h & g \\ h & b & f \\ g & f & c \end{vmatrix} = 0 = \Delta$ (say)

or $\Delta = abc + 2fgh - af^2 - bg^2 - ch^2 = 0$

(2) a circle if $\Delta \neq 0$, $a = b \neq 0$ and $h = 0$.

(3) a parabola if $\Delta \neq 0$ and $h^2 = ab$.

(4) a hyperbola if $\Delta \neq 0$, $h^2 > ab$ and a rectangular hyperbola if $\Delta \neq 0$, $h^2 > ab$ and $a + b = 0$.

(5) an ellipse if $\Delta \neq 0$ and $h^2 < ab$.

PARABOLA

- **Standard equation of parabola :** Let S be the focus, ZM be the directrix and P be the moving point.

  Now, $PM = PS$

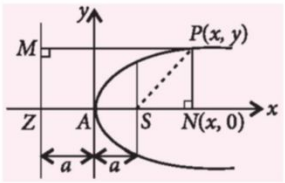
  $\Rightarrow \quad (PM)^2 = (PS)^2$

  $\Rightarrow \quad (PM)^2 = (PN)^2 + (SN)^2$

  $\Rightarrow \quad (PM)^2 = (PN)^2 + (AN - AS)^2$

  $\Rightarrow \quad (x + a)^2 = y^2 + (x - a)^2$

  $\Rightarrow \quad y^2 = 4ax$, which is known as standard equation of parabola.



- **Position of a point w.r.t. parabola :** Let $y^2 = 4ax$ be equation of parabola and $P(x_1, y_1)$ be any point, then
  (i) Point $P(x_1, y_1)$ lies on the parabola if $y_1^2 - 4ax_1 = 0$
  (ii) Point $P(x_1, y_1)$ lies inside the parabola if $y_1^2 - 4ax_1 < 0$
  (iii) Point $P(x_1, y_1)$ lies outside the parabola if $y_1^2 - 4ax_1 > 0$
- **Parametric equations of parabola :** The point $(at^2, 2at)$ satisfies the equation of parabola $y^2 = 4ax$ $\forall$ values of t.

  $\therefore \quad x = at^2$, $y = 2at$ are known as parametric equations of the parabola and $(at^2, 2at)$ are called parametric coordinates.

  Note : If we have the parabola of the form $(y - k)^2 = 4a(x - h)$, then parametric coordinates are $(h + at^2, k + 2at)$ and parametric equations are $x = h + at^2$ and $y = k + 2at$.

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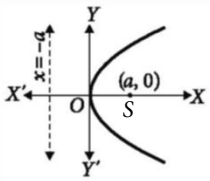
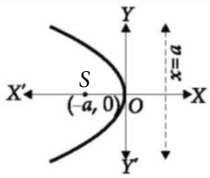
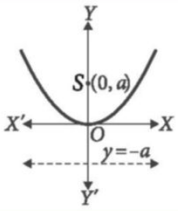
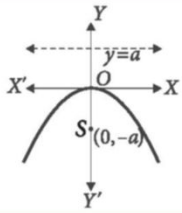
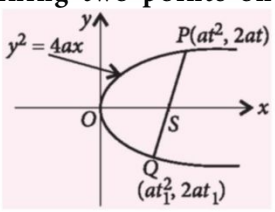
- **Different forms of parabola passing through origin :**

Equation of Parabola	$y^2 = 4ax$	$y^2 = -4ax$	$x^2 = 4ay$	$x^2 = -4ay$
Graph				
Vertex	(0, 0)	(0, 0)	(0, 0)	(0, 0)
Focus	$S(a, 0)$	$S(-a, 0)$	$S(0, a)$	$S(0, -a)$
Equation of axis	$y = 0$	$y = 0$	$x = 0$	$x = 0$
Equation of tangent at vertex (0, 0)	$x = 0$	$x = 0$	$y = 0$	$y = 0$
Equation of directrix	$x = -a$	$x = a$	$y = -a$	$y = a$
Focal distance of a point	$x + a$	$a - x$	$y + a$	$a - y$
Length of latus rectum	$4a$	$4a$	$4a$	$4a$
Ends of latus rectum	$(a, \pm 2a)$	$(-a, \pm 2a)$	$(\pm 2a, a)$	$(\pm 2a, -a)$

- **Double ordinate :** A chord perpendicular to the axis of parabola is called double ordinate.
- **Equation of chord joining two points on the parabola :** Let $P(at^2, 2at)$ and $Q(at_1^2, 2at_1)$ be any two points on the parabola $y^2 = 4ax$.



Then equation of chord PQ passing through these points is $y(t + t_1) = 2x + 2att_1$.

Point of intersection of a line and a parabola, condition of tangency and point of contact

Let $y = mx + c$ ...(i) be equation of line and $y^2 = 4ax$ ...(ii) be the equation of parabola.

$\therefore$ From (i) and (ii), we have

$m^2x^2 + 2(mc - 2a)x + c^2 = 0$ ...(*)

- If $D = 16a(a - mc) > 0$ then line will intersect the parabola at two distinct points and point of intersection are given by

$$\left(\frac{2a - mc \pm 2\sqrt{a(a-mc)}}{m^2}, \frac{2a \pm 2\sqrt{a(a-mc)}}{m}\right)$$

- The line (i) touch the parabola (ii) if both roots of (*) are equal *i.e.*, $D = 0$ *i.e.*, $a(a - mc) = 0$ $\Rightarrow c = \frac{a}{m}$ then line (i) reduces to $y = mx + \frac{a}{m}, m \neq 0$, which is the equation of tangent at point $\left(\frac{a}{m^2}, \frac{2a}{m}\right)$
- The line (i) neither intersect nor touch the parabola $y^2 = 4ax$ if $D < 0$ *i.e.*, $mc > a$.

Tangent drawn at a point to the given parabola

Curve	Point	Equation of Tangent
$y^2 = 4ax$	(x_1, y_1)	$yy_1 = 2a(x + x_1)$
$y^2 = 4ax$	$(at^2, 2at)$	$ty = x + at^2$
$y^2 = 4ax$	$\left(\frac{a}{m^2}, \frac{2a}{m}\right)$	$y = mx + \frac{a}{m}$

Point of intersection of tangents at t_1 and t_2

Equation of tangent at A is

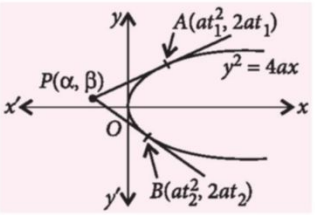
$t_1y = x + at_1^2$ ...(i)

Equation of tangent at B is

$t_2y = x + at_2^2$ ...(ii)

Solving (i) and (ii), we get

$P(\alpha, \beta) = P(at_1t_2, a(t_1 + t_2))$



- **Director circle :** Locus of the point of intersection of perpendicular tangents of the conic is called director circle.

Note : (i) If the tangents at t_1 and t_2 are at right angle, then $t_1t_2 = -1$ so $P(\alpha, \beta) = P(-a, a(t_1 + t_2))$.

(ii) Locus of the point of intersection of perpendicular tangents to the parabola $y^2 = 4ax$ is its directrix *i.e.*, $x = -a$

(iii) Let $A(at_1^2, 2at_1)$, $B(at_2^2, 2at_2)$ and $C(at_3^2, 2at_3)$ be three points lies on the parabola $y^2 = 4ax$, then area of triangle ABC is given by $\Delta = a(t_1 - t_2)(t_2 - t_3)(t_3 - t_1)$.

(iv) Area of triangle formed by the tangents at A, B and C is twice the area of triangle formed by the points A, B and C *i.e.*, $2 \times a(t_1 - t_2)(t_2 - t_3)(t_3 - t_1)$.

Equation of Normals to the Parabola

- Let $P(at^2, 2at)$ be any point on the parabola $y^2 = 4ax$, then

$$\text{slope of tangent} = \frac{2a}{y} \text{ i.e., } \left(\frac{dy}{dx}\right)_P = \frac{2a}{2at} = \frac{1}{t}$$

$\Rightarrow$ Slope of normal at $P = -t$

$\therefore$ Equation of the normal is

$y = -tx + 2at + at^3$ ...(i)

$\Rightarrow$ $y = mx - 2am - am^3$ (by putting $-t = m$)...(ii)

- **Co-normal points :** The points on the curve at which the normal passes through a common point are called co-normal points. For the parabola $y^2 = 4ax$ the equation of normal to parabola is (ii) which passes through $P(x_1, y_1)$.

$\therefore$ $am^3 + m(2a - x_1) + y_1 = 0$ ...(iii)

which is cubic in m will give three values of m. These values may be real or imaginary. If m_1, m_2, m_3 are roots of equation (iii), then we have

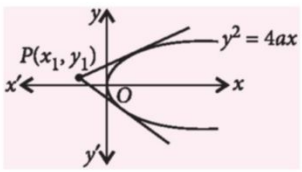
(a) $m_1 + m_2 + m_3 = 0$ *i.e.*, sum of the slopes of concurrent normals at co-normal points is equal to zero *i.e.*, the sum of ordinates of the feet of these normals is $-2a(m_1 + m_2 + m_3) = 0$

(b) $m_1m_2 + m_2m_3 + m_1m_3 = \frac{2a - x_1}{a}$

(c) $m_1m_2m_3 = \frac{-y_1}{a}$

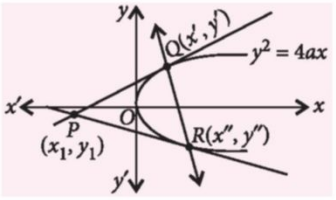
Note : If normal at 't_1' meets the parabola again at 't_2', then $t_2 = -t_1 - \frac{2}{t_1}$

Joint equation of pair of tangents from a point to the parabola



Joint equation of pair of tangents is given by $SS' = T^2$ where $S' = y_1^2 - 4ax_1$ and $T = yy_1 - 2a(x + x_1)$ and $S = y^2 - 4ax$

Equation of chord of contact of tangents to parabola



Equation of chord of contact *i.e.*, QR is $yy_1 - 2a(x + x_1) = 0$

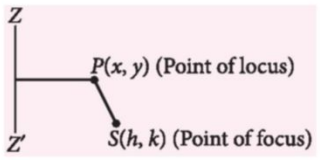
- **Equation of the chord whose middle point is given :** Let $y^2 = 4ax$ be equation of parabola and $P(x_1\ y_1)$ be mid point of chord, then its equation is given by $T = S_1$ or S'

i.e., $yy_1 - 2a(x + x_1) = y_1^2 - 4ax_1$

ELLIPSE

An ellipse is the locus of a point which moves in such a way that its distance from a fixed point to its distance from a fixed straight line bears a constant ratio which is always less than one.

- **General equation of an ellipse :**



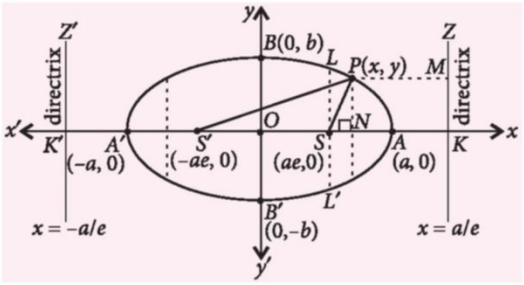
The general equation of an ellipse, whose focus is $S(h, k)$ and directrix is $ax + by + c = 0$ and eccentricity e, is given by

$$\frac{SP}{PM} = e$$

$$\Rightarrow (SP)^2 = e^2(PM)^2$$

$$\Rightarrow (x - h)^2 + (y - k)^2 = e^2\left[\frac{(ax + by + c)^2}{a^2 + b^2}\right] \quad ...(*)$$

- **Standard equation of an ellipse :**



By definition of an ellipse, $\frac{SP}{PM} = e$

$$\Rightarrow (SP)^2 = e^2(PM)^2$$

$$\Rightarrow (x - ae)^2 + (y - 0)^2 = e^2\left(x - \frac{a}{e}\right)^2$$

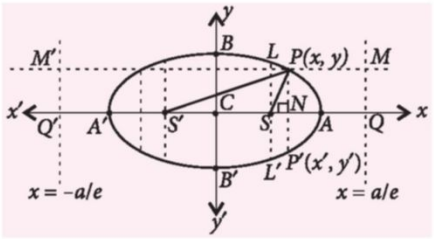
$$\Rightarrow x^2(1 - e^2) + y^2 = a^2(1 - e^2)$$

$$\Rightarrow \frac{x^2}{a^2} + \frac{y^2}{a^2(1 - e^2)} = 1$$

or $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ (where $b^2=a^2(1-e^2)$ and $a>b$) which is the standard equation of the ellipse. In the figure AA' and BB' are called the major and minor axis respectively. The length of major axis = $2a$ and length of minor axis = $2b$. $S(ae, 0)$ and $S'(-ae, 0)$ are called foci of ellipse. ZK and ZK' are two directrices of the ellipse whose equations are $x=\pm\frac{a}{e}$. The point which bisect every chord when passes through it, is called centre. The point at which major and minor axis intersect is called the centre.

- **Position of a point w.r.t. an ellipse :** Let the ellipse be $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1\,(a>b)$ and $P(x_1, y_1)$ be any point, then P will lie outside, on or inside the ellipse according as $\frac{x_1^2}{a^2}+\frac{y_1^2}{b^2}$ >, =, < 1
- **Equation of ellipse in parametric form :** Consider the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ ...(*)

  Any point $P(a\cos\theta, b\sin\theta)$ satisfies the equation of ellipse (*), where θ is an eccentric angle, are parametric co-ordinates and $x = a\cos\theta$, $y = b\sin\theta$ are called parametric equation of ellipse (*).
- **Length of latus rectum :** The length of latus rectum is LSL' which is a line segment passes through the foci and whose end points lies on the ellipse and its length equal to $\frac{2b^2}{a}=2a(1-e^2)$ (Here $a > b$)
- **Focal distances of a point on the ellipse :**



By definition, we have

$\frac{PS}{PM}=e$ and $\frac{PS'}{PM'}=e$ $(e<1$ for both ratio)

$\Rightarrow$ $PS = ePM$ but $PM = NQ$

$\Rightarrow$ $PS = e\,NQ = e(CQ - CN)$

$$=e\left(\frac{a}{e}-x\right)=a-ex \qquad ...(i)$$

and $PS' = ePM' = e(Q'N) = e(Q'C + CN)$

$$=e\left(\frac{a}{e}+x\right)=a+ex \qquad ...(ii)$$

Note : Length of major axis is sum of distances of moving point P to foci. In the figure length of major axis is $AA' = PS + PS' = (a + ex) + a - ex = 2a$

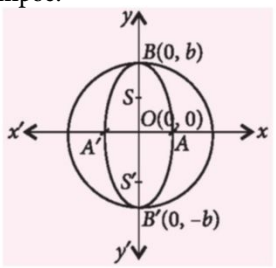
- **Ordinate and double ordinate :** Let P be a point on the ellipse. Draw PN be perpendicular to major axis AA' such that PN produced meet the ellipse at P' then PN is called ordinate of point P and PNP' is called double ordinate of the point P.

Eccentricity of an ellipse $(a > b)$ $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ is given by

$$e^2=\left(1-\frac{b^2}{a^2}\right)=1-\left(\frac{2b}{2a}\right)^2=1-\left(\frac{\text{length of minor axis}}{\text{length of major axis}}\right)^2$$

$$\therefore\quad e=\sqrt{1-\left(\frac{\text{length of minor axis}}{\text{length of major axis}}\right)^2}$$

- **Auxiliary circle of an ellipse :** The circle described on the major axis of an ellipse as diameter is called an auxiliary circle of the ellipse.



If $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1(a<b)$, in this case major axis of the ellipse be along y-axis and length of major axis be $2b$.

$\therefore$ Equation of auxiliary circle is $x^2 + y^2 = b^2$.

- **Equations of tangent to the ellipse :**

(i) **Equation of tangent in point form :** The equation of tangent at any point $P(x_1, y_1)$ to the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ is $\frac{xx_1}{a^2}+\frac{yy_1}{b^2}=1$.

(ii) **Equation of tangent in parametric form :** The equation of tangent to the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ at $P(a\cos\theta, b\sin\theta)$ is

$\frac{x\cos\theta}{a}+\frac{y\sin\theta}{b}=1 \Rightarrow bx\cos\theta + ay\sin\theta = ab.$

(iii) **Equation of tangent to the ellipse in slope form :** The equation of tangent to the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ in slope form is

$y = mx \pm \sqrt{a^2m^2 + b^2}$ and point of contact are $\left(\pm\frac{a^2m}{\sqrt{a^2m^2+b^2}}, \mp\frac{b^2}{\sqrt{a^2m^2+b^2}}\right)$

- **Equation of normal to the ellipse :**

(i) Equation of normal to the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ at $P(x_1, y_1)$ is $\frac{a^2x}{x_1}-\frac{b^2y}{y_1}=a^2-b^2$.

(ii) Equation of normal to the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ at $(a\cos\theta, b\sin\theta)$ in parametric form is $ax\sec\theta - by\operatorname{cosec}\theta = a^2 - b^2$.

(iii) The line $lx + my + n = 0$ will be normal to the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ if $\frac{a^2}{l^2}+\frac{b^2}{m^2}=\frac{(a^2-b^2)^2}{n^2}$.

(iv) Equation of normal to the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ in slope form is $y=mx-\frac{m(a^2-b^2)}{\sqrt{a^2+m^2b^2}}$.

- **Equations of chord of ellipse**

(i) Equation of chord of an ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ bisected at a point $P(x_1, y_1)$ is given by $T = S_1$, where T is equation of tangent to ellipse at $P(x_1, y_1)$ and S_1 is the distance of point $P(x_1, y_1)$ from the ellipse.

$\therefore \quad \frac{xx_1}{a^2}+\frac{yy_1}{b^2}=\frac{x_1^2}{a^2}+\frac{y_1^2}{b^2}$

(ii) Equation of chord joining $P(x_1, y_1)$ and $Q(x_2, y_2)$ to the ellipse is $S_1 + S_2 = S_{12}$, where

$S_1=\frac{xx_1}{a^2}+\frac{yy_1}{b^2}-1$, $S_2=\frac{xx_2}{a^2}+\frac{yy_2}{b^2}-1$

and $S_{12}=\frac{x_1x_2}{a^2}+\frac{y_1y_2}{b^2}-1$

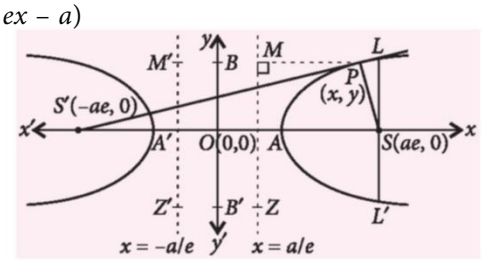
- **Equation of pair of tangents :** Equation of pair of tangents to the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$, from the point $P(x_1, y_1)$ is $SS_1 = T^2$, where $S=\frac{x^2}{a^2}+\frac{y^2}{b^2}-1$, $T=\frac{xx_1}{a^2}+\frac{yy_1}{b^2}-1$ and $S_1=\frac{x_1^2}{a^2}+\frac{y_1^2}{b^2}-1$

i.e., $\left(\frac{x^2}{a^2}+\frac{y^2}{b^2}-1\right)\left(\frac{x_1^2}{a^2}+\frac{y_1^2}{b^2}-1\right)=\left(\frac{xx_1}{a^2}+\frac{yy_1}{b^2}-1\right)^2$

HYPERBOLA

- **Definition (i) :** A hyperbola is the locus of a point which moves in such a way that the ratio of its distance from a fixed point (focus) to its distance from a fixed line (directrix) is always greater than unity.
- **Definition (ii) :** A hyperbola is the set of all points in a plane, the difference of whose distance from two fixed points (foci) is always constant.

i.e., $S'P - SP = 2a$ (constant) (where $S'P = a + ex$ and $SP = ex - a$)



- **Standard equation of hyperbola :** By definition, we have $\frac{SP}{PM}=e \Rightarrow SP^2=e^2(PM)^2$

$\Rightarrow (x-ae)^2+y^2=e^2\left(x-\frac{a}{e}\right)^2$

$\Rightarrow x^2(1 - e^2) + y^2 = a^2(1 - e^2)$

$\Rightarrow \frac{x^2}{a^2}-\frac{y^2}{a^2(e^2-1)}=1$ [As $e>1$ therefore $e^2-1>0$]

$\Rightarrow \frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ [where $b^2=a^2(e^2-1)$]

- **Position of a point w.r.t. hyperbola :** Let $P(x_1, y_1)$ be a point and $\frac{x^2}{a^2}-\frac{y^2}{b^2}-1=0$ be equation of hyperbola, then $P(x_1,y_1)$ will be outside, on or inside the hyperbola according as

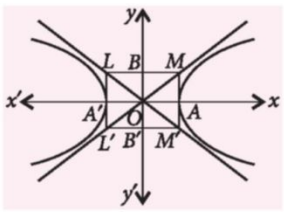
$\frac{x_1^2}{a^2}-\frac{y_1^2}{b^2}-1 <, =,$ or > 0.

Terms of Hyperbola $\left(\frac{x^2}{a^2}-\frac{y^2}{b^2}=1\right)$

- **Transversal axis and conjugate axis :** In the figure, line segment AA' is known as transversal axis and

BB' is called conjugate axis. The length of transversal axis is $2a$ and length of conjugate axis is $2b$. Both axes taken together are called principal axis of the hyperbola. Transversal axis is a line joining the two foci.

- **Centre :** It is middle point of foci $(ae, 0)$, $(-ae, 0)$ *i.e.*, origin $(0, 0)$.
- **Directrices :** Equations of directrices of hyperbola are $x = \frac{a}{e}$ and $x = \frac{-a}{e}$.
- **Vertices :** Point $(a, 0)$ and $(-a, 0)$ are vertices of hyperbola..
- **Latus rectum :** The distance between $L\left(ae, \frac{b^2}{a}\right)$ and $L'\left(ae, -\frac{b^2}{a}\right)$ *i.e.*, LL' is length of latus rectum given by $LL' = \frac{2b^2}{a}$.
- **Conjugate hyperbola :** A hyperbola whose conjugate axis and transversal axis respectively the transversal axis and conjugate axis of the other and vice versa. If $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ is one hyperbola, then its conjugate hyperbola is $\frac{x^2}{a^2} - \frac{y^2}{b^2} = -1$.
- **Asymptotes:** If the length of perpendicular drawn from a point on the hyperbola to a straight line tends to zero as the point moves to infinity, the straight line is called asymptotes of the hyperbola. The equation of asymptotes of hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ are $\frac{x}{a} + \frac{y}{b} = 0$ and $\frac{x}{a} - \frac{y}{b} = 0$.



If we draw lines through the vertices parallel to conjugate axis and through B and B' parallel to transversal axis, then $M(a, b)$, $M'(a, -b)$, $L'(-a, -b)$ and $L(-a, b)$ all lie on the asymptotes $\frac{x^2}{a^2} = \frac{y^2}{b^2}$ *i.e.*, $x = \pm\frac{a}{b}y$ are diagonals of the rectangle $MM'L'L$ which is associated rectangle of the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$.

Note : (i) The slope of asymptotes of $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ are $m_1 = \frac{b}{a}, m_2 = \frac{-b}{a}$.

(ii) The asymptotes of a hyperbola and its conjugate are same.

(iii) The equation of asymptotes of hyperbola $xy = ax + by$ are given by $x = b$, $y = a$.

Line and Hyperbola

- Let $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ be hyperbola and the line $y = mx + c$ be the equation of tangent to the hyperbola if $D = 0$ *i.e.*, $c^2 = a^2m^2 - b^2$

  The line $y = mx + c$ will intersect if $D > 0$ *i.e.*, $c^2 - a^2m^2 + b^2 > 0 \Rightarrow c^2 > a^2m^2 - b^2$

  The line $y = mx + c$ will not intersect *i.e.*, lies outside the hyperbola if $D < 0$ *i.e.*, $c^2 - a^2m^2 + b^2 < 0 \Rightarrow c^2 < a^2m^2 - b^2$.

Tangent and Normal of Hyperbola

- Let $P(x_1, y_1)$ be any point on the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$, the equation of tangent is given by $\frac{xx_1}{a^2} - \frac{yy_1}{b^2} = 1$.
- Equation of tangent to the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ at $P(a\sec\theta, b\tan\theta)$ is given by $\frac{x\sec\theta}{a} - \frac{y\tan\theta}{b} = 1$.
- Equation of pair of tangents from a point $P(x_1, y_1)$ to the hyperbola is given by $SS_1 = T^2$, where

  $S = \frac{x^2}{a^2} - \frac{y^2}{b^2} - 1$, $S_1 = \frac{x_1^2}{a^2} - \frac{y_1^2}{b^2} - 1$,

  $T = \frac{xx_1}{a^2} - \frac{yy_1}{b^2} - 1$.
- Equation of any tangent to $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ in slope form is $y = mx \pm \sqrt{a^2m^2 - b^2}$

Rectangular Hyperbola

A hyperbola is said to be rectangular hyperbola if the asymptotes of the hyperbola are at right angle. The equation of rectangular/equilateral hyperbola is given by $x^2 - y^2 = a^2$.

- As $a^2 = a^2(e^2 - 1) \Rightarrow 1 = e^2 - 1 \Rightarrow e^2 = 2$

  $\Rightarrow e = \sqrt{2}$ *i.e.*, eccentricity of equilateral hyperbola is $\sqrt{2}$.

- When we rotate the axis of hyperbola $x^2 - y^2 = a^2$ through an angle $\frac{\pi}{4}$ about the same origin, the equation of the hyperbola reduces to $xy = \frac{a^2}{2}$ whose asymptotes are coordinate axes *i.e.*, $xy = 0$.
- The parametric representation of $xy = c^2$ $\left(\text{where } c^2 = \frac{a^2}{2}\right)$ is $x = ct, y = \frac{c}{t}$ $[t \in R - \{0\}]$
- The conjugate hyperbola of $xy = c^2$ is $xy = -c^2$.

PROBLEMS

Single Correct Answer Type

1. The tangent to the parabola $y = x^2 - 5$ at a point $A(1, -4)$ touches the circle $x^2 + y^2 - 6x + 10y + K = 0$ then K is

(a) 49 (b) 29
(c) 39 (d) 34

2. The minimum distance of a point on the parabola $y = x^2 - 6$ from the origin is

(a) $\frac{23}{4}$ (b) $\frac{\sqrt{23}}{4}$ (c) $\frac{\sqrt{23}}{2}$ (d) $\sqrt{\frac{23}{2}}$

3. The maximum number of common normals of $y^2 = 4ax$ and $x^2 = 4by$ may be equal to

(a) 5 (b) 4
(c) 3 (d) 2

4. The three normals (each different) are drawn from the point (h, k) to the parabola $y^2 = 8x$ then range of h is

(a) $1 < h < \infty$ (b) $2 < h < \infty$
(c) $3 < h < \infty$ (d) $4 < h < \infty$

5. The length of the chord of the ellipse $\frac{x^2}{25} + \frac{y^2}{16} = 1$ whose middle point is $\left(\frac{1}{2}, \frac{2}{5}\right)$ is

(a) $\frac{\sqrt{41}}{3}$ (b) $\frac{7\sqrt{41}}{5}$
(c) $\frac{3\sqrt{41}}{5}$ (d) None of these

6. The eccentricity of the hyperbola $x^2 - 3y^2 + 1 = 0$ is

(a) $\frac{1}{2}$ (b) 1 (c) 2 (d) 3

7. The eccentricity of the hyperbola whose length of latus rectum is 8 and length of its conjugate axis is equal to half of the distance between its foci, is

(a) $\sqrt{3}$ (b) $\frac{2}{\sqrt{3}}$ (c) $\frac{4}{\sqrt{3}}$ (d) $\frac{4}{3}$

8. If the tangent at the point (h, k) to the curve $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ meet the circle $x^2 + y^2 = a^2$ at the points (x_1, y_1) and (x_2, y_2) then y_1, k, y_2 are in

(a) A.P. (b) G.P.
(c) H.P. (d) None of these

9. Two tangents are drawn from a point on the hyperbola $x^2 - y^2 = a^2 - b^2$ to the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$. If the tangents make the angles α and β with x-axis then the value of $\alpha + \beta$ is

(a) $\frac{\pi}{6}$ (b) $\frac{\pi}{4}$ (c) $\frac{\pi}{3}$ (d) $\frac{\pi}{2}$

10. A hyperbola passes through the point $A(\sqrt{2}, \sqrt{3})$ and has foci at $B(\pm 2, 0)$ then the tangent to this hyperbola at A also passes through the point

(a) $(\sqrt{3}, \sqrt{2})$ (b) $(-\sqrt{2}, -\sqrt{3})$
(c) $(2\sqrt{2}, 3\sqrt{3})$ (d) $(3\sqrt{2}, 2\sqrt{3})$

11. Let $S_1 = x^2 + y^2 - 8x = 0$ and $S_2 = \frac{x^2}{9} - \frac{y^2}{4} - 1 = 0$ and $S_1 = 0$ and $S_2 = 0$ intersect at A and B then equation of circle whose diameter is AB, is

(a) $x^2 + y^2 + 12x + 24 = 0$ (b) $x^2 + y^2 - 16x + 24 = 0$
(c) $x^2 + y^2 + 16x + 24 = 0$ (d) $x^2 + y^2 - 12x + 24 = 0$

12. A hyperbola whose transverse axis is along the major axis of the conic $\frac{x^2}{3} + \frac{y^2}{4} = 4$ and has its vertices at the foci of this conic. If the eccentricity of the hyperbola is $\frac{3}{2}$, then which of the following points does not lie on the curve?

(a) $(\sqrt{5}, 2\sqrt{2})$ (b) $(\sqrt{20}, \sqrt{20})$
(c) $(5\sqrt{2}, 2\sqrt{11})$ (d) $(3\sqrt{5}, 4\sqrt{2})$

13. If tangent to the parabola $y = x^2 + 6$ at $(2, 10)$ touches the circle $x^2 + y^2 + 8x - 2y + K = 0$ (for some fixed K) at a point (α, β) then which of the following is false?

(a) $\alpha = \frac{-8}{17}, \beta = \frac{2}{17}$
(b) Equation of tangent to parabola is $4x - y + 2 = 0$.
(c) Equation of tangent to parabola is $2x - y + 4 = 0$.
(d) The radius of the circle is $\frac{15}{\sqrt{17}}$.

More Than One Correct Answer Type

14. Let P be the point on the parabola $y^2 = 8x$ which is at the least distance from the centre C of the circle

$x^2 + y^2 - 8x - 32y + 256 = 0$. If Q be the point on the circle dividing the line segment CP internally then

(a) x intercept of the normal at P is 12.

(b) length of $QP = 4(\sqrt{5} - 1)$.

(c) equation of tangent at P is $x - 2y + 8 = 0$.

(d) slope of tangent to the circle at $Q = 1/2$.

15. If tangents to the hyperbola $x^2 - 16y^2 = 16$ are drawn from a point (4, 3), then

(a) Equation of tangent is $5x - 12y + 16 = 0$.

(b) Equation of tangent is $x = 4$.

(c) Centroid of the triangle formed by tangents and chord of contact w.r.t. the point (4, 3) is $\left(1, \frac{3}{4}\right)$.

(d) Area of the triangle formed by tangents and the chord of contact is $\frac{27}{2}$ sq. units.

16. If $2x - y + 1 = 0$ is tangent to the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{16} = 1$, then which of the following cannot be the sides of a right angled triangle?

(a) $a, 4, 2$ (b) $a, 4, 1$ (c) $2a, 8, 1$ (d) $2a, 4, 1$

17. The equation of tangent to the ellipse $\frac{x^2}{25} + \frac{y^2}{9} = 1$, which cut equal intercepts on the axes is/are

(a) $y = x + \sqrt{34}$ (b) $y = -x + \sqrt{34}$

(c) $y = -x - \sqrt{34}$ (d) $y = x - \sqrt{34}$

18. The eccentric angle of a point on the ellipse $3x^2 + 5y^2 = 15$ at distance 2 units from the origin is

(a) $\frac{\pi}{4}$ (b) $\frac{3\pi}{4}$ (c) $\frac{5\pi}{4}$ (d) $\frac{7\pi}{4}$

Matrix Match Type

19. Match the following :

	Column-I		Column-II
A.	If e be the eccentricity of the hyperbola $36x^2 + 288x - 64y^2 - 128y - 1792 = 0$ then the value of $4e$ equals	p.	7
B.	The number of values of K, such that the line $y = 4x + K$ touches the curve $x^2 + 4y^2 = 4$ is	q.	3
C.	The eccentricity of an ellipse with centre at $O(0, 0)$ is $\frac{1}{3}$. If one directrix is $x = 6$ and ellipse reduces to $K_1x^2 + K_2y^2 = K_3$ then $\frac{K_3 - K_1 - K_2}{5} =$	r.	5
D.	If the foci of an ellipse and the hyperbola whose equations are respectively $\frac{x^2}{16} + \frac{y^2}{b^2} = 1$ and $\frac{25x^2}{144} - \frac{25y^2}{81} = 1$, coincide with each other, then the value of b^2 equals	s.	2

Numerical Answer Type

20. A variable chord AB of the parabola $y = 6x^2$ subtends 90° angle at the vertex. If the locus of the point of intersection of normals at A and B is $y = lx^2 + m$ $(l, m \in R)$ then value of $l^2 + 27m$ is

21. If normal at an extremity of a latus rectum of an ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ $(a > b)$ passes through one extremity of its minor axis then the value of $20(2e^2 + 1)^2$ is

22. If e_1 be the eccentricity of the hyperbola whose asymptotes are $5x + 3y - 7 = 0$ and $3x - 5y + 12 = 0$ and e_2 be the eccentricity of hyperbola whose asymptotes are $12x - 5y + 5 = 0$ and $5x + 12y - 7 = 0$ then value of $16\sqrt{2}(e_1^3 e_2^4)$ is

23. The equation of the hyperbola whose asymptotes are $2x + y + 3 = 0$ and $3x + 4y + 5 = 0$ passes through the point $(-1, 1)$ is $6x^2 + 4y^2 + 11xy + 19x + 17y + \lambda = 0$ then the equation of the conjugate hyperbola is $6x^2 + 4y^2 + 11xy + lx + my + n = 0$ then the value of $(l - m + n)$ is

SOLUTIONS

1. (b) : Equation of tangent to the parabola $y = x^2 - 5$ at $A(1, -4)$ is given by $2x - y - 6 = 0$ ...(i)

(i) will touch the circle $x^2 + y^2 - 6x + 10y + K = 0$ whose centre is $(3, -5)$ and radius $= \sqrt{34 - K}$

$\therefore \quad \sqrt{34 - K} = \frac{|2(3) - (-5) - 6|}{\sqrt{5}} \Rightarrow K = 29$

2. (c) : Let (α, β) be any point on the parabola

$\therefore \quad (\alpha, \beta) = (\alpha, \alpha^2 - 6)$

$\therefore$ Distance (d) between $(0, 0)$ and $(\alpha, \alpha^2 - 6)$ is $\sqrt{(\alpha - 0)^2 + (\alpha^2 - 6)^2}$

$\Rightarrow \quad d^2 = \alpha^4 - 11\alpha^2 + 36 \Rightarrow d = \sqrt{(\alpha^2)^2 - 11\alpha^2 + 36}$

$\therefore \quad \frac{d}{d\alpha}(d^2) = 4\alpha^3 - 22\alpha = 2\alpha(2\alpha^2 - 11)$

For minima and maxima put $\frac{d(d^2)}{d\alpha} = 0$.

$\Rightarrow \quad 2\alpha(2\alpha^2 - 11) = 0 \quad \Rightarrow \quad \alpha = 0, \text{ or } \alpha^2 = \frac{11}{2}$

Again $\frac{d^2(d^2)}{d\alpha^2} = 12\alpha^2 - 22$

When $\alpha = 0$, $\frac{d^2(d^2)}{d\alpha^2} = -22 < 0$

When $\alpha^2 = \frac{11}{2}$, $\frac{d^2d^2}{d\alpha^2} = 44 > 0$

So, distance (d) will be minimum when $\alpha^2 = \frac{11}{2}$.

$\therefore \quad d(\text{min. distance}) = \sqrt{\left(\frac{11}{2}\right)^2 - 11 \times \frac{11}{2} + 36} = \frac{\sqrt{23}}{2}$

3. (a)

4. (d) : Equation of normal at $(at^2, 2at)$ to the parabola $y^2 = 4ax$ is given by $y + xt = at^3 + 2at$

Now parabola is $y^2 = 8x = 4(2)x \quad \therefore \quad a = 2$

Equation of normal at $(2t^2, 4t)$ is

$y + xt = 2t^3 + 4t$ which passes through (h, k)

$\therefore \quad 2t^3 + 4t = k + ht \Rightarrow 2t^3 + t(4 - h) - k = 0 \quad ...(i)$

Let roots of (i) be t_1, t_2, t_3

$\therefore \quad t_1 + t_2 + t_3 = 0$ and $t_1t_2 + t_2t_3 + t_3t_1 = \frac{4-h}{2}$

$\Rightarrow \quad t_3 = -(t_1 + t_2)$ and $t_1t_2 + (t_1 + t_2)t_3 = \frac{4-h}{2}$

$\therefore \quad \frac{4-h}{2} = t_1t_2 - (t_1 + t_2)^2$

$\Rightarrow \quad \frac{h-4}{2} = (t_1 + t_2)^2 - t_1t_2 \Rightarrow \frac{h-4}{2} = t_1^2 + t_2^2 + t_1t_2$

$\Rightarrow \quad \frac{h-4}{2} = \left(t_1 + \frac{t_2}{2}\right)^2 + \frac{3}{4}t_2^2 > 0$

$\Rightarrow \quad h > 4 \quad \Rightarrow \quad h \in (4, \infty)$

5. (b) : We have, $\frac{x^2}{25} + \frac{y^2}{16} = 1 \quad ...(i)$

Equation of chord with middle point $\left(\frac{1}{2}, \frac{2}{5}\right)$ is given by $T = S_1$

i.e., $\frac{\frac{1}{2}x}{25} + \frac{\frac{2}{5}y}{16} - 1 = \left(\frac{1}{4}\right)\frac{1}{25} + \left(\frac{4}{25}\right)\frac{1}{16} - 1$

$\Rightarrow \quad 5y = 4(1 - x) \quad ...(ii)$

Solving (i) and (ii), we get

$4x^2 + 4(1 - x)^2 = 100 \Rightarrow x^2 + (1 - x)^2 = 25$

$\Rightarrow \quad (x - 4)(x + 3) = 0 \quad \Rightarrow \quad x = 4, -3$

When $x = 4$ then $y = -\frac{12}{5}$ and when $x = -3$, $y = \frac{16}{5}$

$\therefore \quad$ Required length $= \sqrt{(4 + 3)^2 + \left(\frac{28}{5}\right)^2} = \frac{7\sqrt{41}}{5}$

6. (c)

7. (b) : Let the equation of hyperbola is $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

Given, length of latus rectum $= \frac{2b^2}{a} = 2a(e^2 - 1) = 8$

According to question, $2b = \frac{1}{2}(2ae)$

$\Rightarrow \quad 2b = ae \quad \Rightarrow \quad 4b^2 = a^2e^2$

Now, $b^2 = a^2(e^2 - 1)$

$\Rightarrow \quad 4b^2 = 4a^2(e^2 - 1) \quad \therefore \quad 4a^2(e^2 - 1) = a^2e^2$

$\Rightarrow \quad 3e^2 = 4 \quad \Rightarrow \quad e^2 = \frac{4}{3} \quad \Rightarrow e = \frac{2}{\sqrt{3}} \text{ (as } e > 0)$

8. (c) : Given curve is $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1 \quad ...(i)$

$\therefore \quad$ Tangent to (i) at (h, k) is $\frac{xh}{a^2} - \frac{yk}{b^2} = 1$

$\Rightarrow \quad xh = a^2\left(1 + \frac{yk}{b^2}\right) \quad \Rightarrow \quad x = \frac{a^2}{h}\left(1 + \frac{yk}{b^2}\right) \quad ...(ii)$

Using (ii) in equation of circle $x^2 + y^2 = a^2$, we get

$\frac{a^4}{h^2}\left(1 + \frac{yk}{b^2}\right)^2 + y^2 = a^2$

$\Rightarrow \quad (b^2 + yk)^2 + \frac{h^2b^4y^2}{a^4} = \frac{h^2b^4}{a^2}$

$\Rightarrow \quad \left(k^2 + \frac{h^2b^4}{a^4}\right)y^2 + 2b^2ky + b^4\left(1 - \frac{h^2}{a^2}\right) = 0$

$\Rightarrow \quad \left(k^2 + \frac{h^2b^4}{a^4}\right)y^2 + 2b^2ky - b^2k^2 = 0$

$\left(\text{Using } \frac{h^2}{a^2} - \frac{k^2}{b^2} = 1\right)$

Which is quadratic in y. Let its roots are y_1 and y_2

$\therefore \quad y_1 + y_2 = \frac{-2b^2k}{\left(k^2 + \frac{h^2b^4}{a^4}\right)}$ and $y_1y_2 = \frac{-b^2k^2}{k^2 + \frac{h^2b^4}{a^4}}$

Now, $\frac{y_1 + y_2}{y_1y_2} = \frac{2bk^2}{b^2k^2} = \frac{2}{k} \quad \Rightarrow \quad k = \frac{2y_1y_2}{y_1 + y_2}$

$\Rightarrow \quad y_1, k, y_2$ are in H.P.

9. (d) : Equation of tangent to the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ is given by $y = mx + \sqrt{a^2m^2 + b^2}$

$\Rightarrow \quad (y - mx)^2 = a^2m^2 + b^2$

$\Rightarrow \quad m^2(x^2 - a^2) - 2xym + (y^2 - b^2) = 0$ ...(i)

Let m_1 and m_2 are roots of (i) then

$$m_1 + m_2 = \frac{2xy}{x^2 - a^2} \text{ and } m_1m_2 = \frac{y^2 - b^2}{x^2 - a^2} \quad ...(ii)$$

$\therefore$ Tangent meet at a point on the hyperbola.

$x^2 - y^2 = a^2 - b^2 \Rightarrow y^2 - b^2 = x^2 - a^2$

$$\Rightarrow \quad \frac{y^2 - b^2}{x^2 - a^2} = 1 \quad ...(iii)$$

From (ii) and (iii), we get $m_1m_2 = 1$

$\Rightarrow \tan\alpha \cdot \tan\beta = 1 \Rightarrow \cot\beta = \tan\alpha$

$$\Rightarrow \tan\alpha = \tan\left(\frac{\pi}{2} - \beta\right) \Rightarrow \alpha = \frac{\pi}{2} - \beta \Rightarrow \alpha + \beta = \frac{\pi}{2}$$

10. (c) **11. (d)**

12. (d) : Given conic is $\frac{x^2}{12} + \frac{y^2}{16} = 1$ which is an ellipse whose y-axis is major axis and foci lies on y-axis.

Also foci are $(0, ae)$, $(0, -ae)$ where $a = 4$ and $b = 2\sqrt{3}$

$$\therefore \quad e = \sqrt{1 - \frac{b^2}{a^2}} = \sqrt{1 - \frac{12}{16}} = \frac{1}{2}$$

Now, foci $\equiv (0, \pm ae) \equiv (0, 2)$ and $(0, -2)$

Now, transverse axis of required hyperbola is y-axis whose length $2a = 4 \Rightarrow a = 2$

Now, value of b for required hyperbola whose eccentricity $e = 3/2$is

$$b^2 = a^2(e^2 - 1) \Rightarrow b^2 = 4\left(\frac{9}{4} - 1\right) \Rightarrow b^2 = 5$$

Required hyperbola is $\frac{x^2}{5} - \frac{y^2}{4} = -1$

The point $(3\sqrt{5}, 4\sqrt{2})$ does not satisfy $\frac{x^2}{5} - \frac{y^2}{4} = -1$

13. (c) : Equation of tangent to $y = x^2 + 6$ at $(2, 10)$ is

$$\frac{1}{2}(y + 10) = 2x + 6 \Rightarrow 4x - y + 2 = 0 \quad ...(i)$$

Now, equation of circle is $x^2 + y^2 + 8x - 2y + K = 0$

(i) will touch the circle if $\perp$ distance from centre $(-4, 1)$ to the line $4x - y + 2 = 0$ is equal to the radius of the circle.

$$\therefore \quad \sqrt{(-4)^2 + 1 - K} = \frac{|-16 - 1 + 2|}{\sqrt{(-4)^2 + 1^2}} \Rightarrow K = \frac{64}{17}$$

$\therefore$ Radius of circle $= \frac{15}{\sqrt{17}}$

Now, (i) touches the circle $x^2 + y^2 + 8x - 2y + \frac{64}{17} = 0$ at (α, β) which is the coordinates of foot of $\perp$ drawn from centre $(-4, 1)$ to the tangent line

$$\therefore \quad \frac{\alpha + 4}{4} = \frac{\beta - 1}{-1} = -\frac{(-16 - 1 + 2)}{4^2 + (-1)^2} = \frac{15}{17}$$

$$\therefore \quad (\alpha, \beta) = \left(\frac{60}{17} - 4, \frac{-15}{17} + 1\right) = \left(\frac{-8}{17}, \frac{2}{17}\right)$$

14. (a, b, c, d) : Equation of circle is

$x^2 + y^2 - 8x - 32y + 256 = 0$

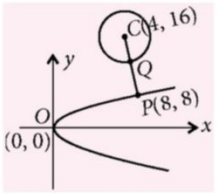
$\Rightarrow \quad (x - 4)^2 + (y - 16)^2 = 4^2$

$\therefore$ Centre $C \equiv (4, 16)$ and radius $R = 4$

Now, equation of normal to the parabola $y^2 = 8x$ is

$y = mx - 4m - 2m^3$ ...(i)

For the least distance from the centre C to parabola the normal must pass through the centre $(4, 16)$ of circle



$\therefore \quad 16 = 4m - 4m - 2m^3$

$\Rightarrow \quad m^3 = (-2)^3 \Rightarrow m = -2$

$\therefore$ Equation of normal is $y = -2x + 24$ (Using (i))

$\therefore$ Slope of tangent at $Q = 1/2$.

Also, x intercept of normal at P is obtained by putting $y = 0$ in (i), $x = 12$

Now, any point on the parabola $y^2 = 4ax$ is $(am^2, -2am)$ $= (8, 8)$ (using $a = 2$, $m = -2$)

$\therefore$ Equation of tangent to parabola at P is

$$y - 8 = \frac{1}{2}(x - 8) \Rightarrow x - 2y + 8 = 0$$

NTA Declares JEE Main January 2019 NTA Score

The first JEE (Main) Examination was conducted by NTA between 8th and 12th January, 2019 in two shifts per day across 258 cities in the country and abroad. 15 candidates have received 100 NTA Score in January 2019 examination in Paper-I (B.E./B.Tech.). State wise toppers and their NTA Scores in Paper-1 (B.E./B.Tech.) can be checked from the official website of JEE main.

- NTA Scores are normalised scores across multi session papers and are based on the relative performance of all those who appeared for the examination in one session. The marks obtained are converted into a scale ranging from 100 to 0 for each session of examinees.
- The NTA Score indicates the percentage of candidates that have scored EQUAL TO OR BELOW (same or lower raw scores) that particular Percentile in that session.
- The Percentile scores of a Candidate have been calculated as follows : (100 × Number of candidates appeared in the 'Session' with raw score EQUAL TO OR LESS than the candidate) ÷ (Total number of the candidates appeared in the Session)
- The NTA Score of the Total is NOT an aggregate or average of the NTA Score of individual subjects. NTA score is not the same as percentage of marks obtained.
- After the JEE MAIN APRIL-2019 Exam, the Ranks of the candidates will be released taking into consideration better of the two NTA Scores of all candidates appeared in January 2019 and April 2019 examinations and in accordance with the policy already made available on the website https://jeemain.nic.in.
- The next online application for JEE Main April 2019 Examination shall start from 8th February and shall remain open till 7th March 2019. The uploading of images and payment of the application fee will be up to 8th March 2019. The JEE (Main) April 2019 Examination will be conducted in CBT mode between 6th and 20th April, 2019.

Now $PQ = CP - CQ = CP$ – radius of circle (R)

$= 4\sqrt{5} - 4 = 4(\sqrt{5} - 1)$

15. (a, b, c, d) : Given hyperbola is

$x^2 - 16y^2 = 16 \Rightarrow \frac{x^2}{16} - \frac{y^2}{1} = 1 = \frac{x^2}{a^2} - \frac{y^2}{b^2} (a > b)$

where $a = 4$, $b = 1$

Equation of tangents to the given hyperbola is

$y = mx \pm \sqrt{16m^2 - 1}$ ...(i)

$\because$ Eq. (i) passes through (4, 3)

$\therefore \quad 3 = 4m \pm \sqrt{16m^2 - 1} \Rightarrow (3 - 4m)^2 = 16m^2 - 1$

$\Rightarrow \quad 9 - 24m + 16m^2 = 16m^2 - 1$

$\Rightarrow \quad m_1 = \frac{5}{12}$ and $m_2 =$ not defined *i.e.* ∞

$\therefore$ Equation of tangents through the point (4, 3) with slopes m_1 and m_2 are

$y - 3 = \frac{5}{12}(x - 4)$ and $y - 3 = \frac{1}{0}(x - 4)$

$\Rightarrow \quad 5x - 12y + 16 = 0$ ...(ii) and $x = 4$ ...(iii)

Again the equation of chord of contact to the hyperbola w.r.t. the point (4, 3) is

$x(4) - 16y(3) = 16$ or $x - 12y = 4$ ...(iv)

First, we find vertices of triangles using (ii), (iii) and (iv) which are given by (4, 3), (4, 0) and $\left(-5, \frac{-3}{4}\right)$.

The centroid of Δ formed with these points is given by

$$\left(\frac{4 + 4 - 5}{3}, \frac{3 + 0 - 3/4}{3}\right) = \left(1, \frac{3}{4}\right)$$

and its area is $\frac{1}{2}\begin{vmatrix} 4 & 3 & 1 \\ 4 & 0 & 1 \\ -5 & -3/4 & 1 \end{vmatrix} = \frac{27}{2}$ sq. units

16. (a, b, c) : The line $2x - y + 1 = 0$ be tangent to the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{16} = 1$ if both roots of the equation

$\frac{x^2}{a^2} - \frac{(2x + 1)^2}{16} = 1$ are equal

i.e., discriminant of $(16 - 4a^2)x^2 - 4a^2x - 17a^2 = 0$

$\Rightarrow \quad 16a^4 = 4(-17a^2)(16 - 4a^2) \Rightarrow a = \sqrt{17}/2$

Thus, we have set of numbers

$\left(\frac{\sqrt{17}}{2}, 4, 2\right); \left(\frac{\sqrt{17}}{2}, 4, 1\right); (\sqrt{17}, 8, 1)$ and $(\sqrt{17}, 4, 1)$

Only set $(\sqrt{17}, 4, 1)$ represents the sides of a right angled triangle.

17. (a, b, c, d) : Equation of tangent to the ellipse

$y = mx \pm \sqrt{a^2m^2 + b^2}$

$\because$ Tangent cuts equal intercept on the axes

$\therefore$ Slopes are 45° and 135° $\Rightarrow \quad m = \pm 1$

$(\because a^2 = 25, b^2 = 9)$

$\therefore$ Equation of tangents are $y = \pm x \pm \sqrt{25 + 9}$

$\Rightarrow \quad y = \pm x \pm \sqrt{34}$

18. (a, b, c, d) : We have, $\frac{x^2}{5} + \frac{y^2}{3} = 1$ ...(i)

$\therefore$ The point (P) on the ellipse is $(\sqrt{5}\cos\theta, \sqrt{3}\sin\theta)$, where θ is eccentric angle

$\because$ Given, distance $OP = 2$ units (where O is origin)

$\therefore \quad 5\cos^2\theta + 3\sin^2\theta = (2)^2 \Rightarrow \cos 2\theta = 0$

$\therefore \quad 2\theta = (2n + 1)\frac{\pi}{2}, \ (n \in I) \Rightarrow 2\theta = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \frac{7\pi}{2}$

$\theta = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$

19. A → r, B → s, C → q, D → p

(A) $36x^2 + 288x - 64y^2 - 128y - 1792 = 0$

$\Rightarrow \quad 36(x + 4)^2 - 64(y + 1)^2 = 2304$

$\Rightarrow \quad \frac{(x + 4)^2}{64} - \frac{(y + 1)^2}{36} = 1 \Rightarrow a^2 = 64, b^2 = 36$

$\therefore \quad e = \sqrt{1 + \frac{36}{64}} = \frac{10}{8} = \frac{5}{4} \Rightarrow 4e = 5$

(B) Given curve is $x^2 + 4y^2 = 4 \Rightarrow \frac{x^2}{4} + \frac{y^2}{1} = 1$

It is an ellipse where $a^2 = 4$, $b^2 = 1$ and the line $y = mx + K$ will be tangent if $K^2 = a^2m^2 + b^2$

$\therefore \quad K^2 = 4 \cdot 4^2 + 1 \Rightarrow K = \pm\sqrt{65}$

$\therefore$ Number of values of $K = 2$.

(C) Equation of directrix $x = \frac{a}{e} \Rightarrow a = xe \Rightarrow a = 2$

Now, $b^2 = a^2(1 - e^2) = 4\left(1 - \frac{1}{9}\right) = \frac{4 \times 8}{9} = \frac{32}{9}$

$\therefore$ The equation of ellipse is $\frac{x^2}{4} + \frac{9y^2}{32} = 1$

$\Rightarrow \quad 8x^2 + 9y^2 = 32 \Rightarrow K_1x^2 + K_2y^2 = K_3$

$\therefore \quad \frac{K_3 - K_1 - K_2}{5} = \frac{32 - 8 - 9}{5} = 3$

(D) For hyperbola, we have

$e^2 = \frac{a^2 + b^2}{a^2} = \frac{144 + 81}{144} = \frac{225}{144} \Rightarrow e = \frac{5}{4}$

and $a^2 = \frac{144}{25} \Rightarrow a = \frac{12}{5}$

$\therefore$ Foci are $(\pm ae, 0) = \left(\pm \frac{12}{5} \times \frac{5}{4}, 0\right) = (\pm 3, 0)$

Now, for ellipse ($ae = 3$) (foci of an ellipse and hyperbola coincide with each other), $a = 4$

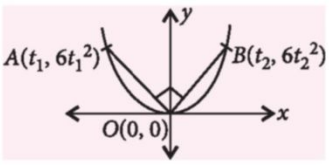
Now, $b^2 = a^2(1 - e^2) = a^2 - a^2e^2 = 16 - 9 = 7$

20. (45) : Let $A(t_1, 6t_1^2)$ and $B(t_2, 6t_2^2)$ be two points on the parabola $y = 6x^2$

$\therefore$ Slope of $OA = \frac{6t_1^2}{t_1} = 6t_1$ and slope of $OB = 6t_2$

Now OA and OB are mutually $\perp$ to each other

i.e. $36t_1t_2 = -1$



Now equations of normals at A and B are

$$y - 6t_1^2 = -\frac{1}{6t_1}(x - t_1) \Rightarrow x + 6t_1y = t_1 + 36t_1^3 \quad ...(i)$$

and $$y - 6t_2^2 = -\frac{1}{6t_2}(x - t_2) \Rightarrow x + 6t_2y = t_2 + 36t_2^3 \quad ...(ii)$$

Let (h, k) be the locus of point of intersection of normals therefore (h, k) satisfies equation (i) and (ii) Thus, we have

$h + 6t_1k = t_1 + 36t_1^3 \Rightarrow ht_2 + 6t_1t_2k = t_1t_2 + 36t_1^3t_2$

and $h + 6t_2k = t_2 + 36t_2^3 \Rightarrow ht_1 + 6t_2t_1k = t_2t_1 + 36t_2^3t_1$

Solving above equations for h and k we get

$h(t_2 - t_1) = 36t_1t_2(t_1^2 - t_2^2)$

$\Rightarrow h = t_1 + t_2 \quad ...(iii)$

and $6k(t_1 - t_2) = (t_1 - t_2) + 36(t_1 - t_2)(t_1^2 + t_1t_2 + t_2^2)$

$\Rightarrow 6k = 1 + 36(t_1^2 + t_1t_2 + t_2^2)$

$= 1 + 36(t_1 + t_2)^2 - 36t_1t_2$

$= 36(t_1 + t_2)^2 + 2 \quad (\because 36t_1t_2 = -1)$

$\Rightarrow 6k = 36h^2 + 2$ (Using (iii))

$\Rightarrow k = 6h^2 + \frac{1}{3}$

$\therefore$ Locus of (h, k) is

$y = 6x^2 + \frac{1}{3} = lx^2 + m \quad \therefore l = 6, m = \frac{1}{3}$

$\therefore l^2 + 27m = 36 + 9 = 45$

21. (100) : As $a > b$ therefore minor axis of the ellipse is y-axis.

Let LL' be the latus rectum of ellipse (you can select L_1L_1' also)

If we draw tangent at $L'\left(ae, -\frac{b^2}{a}\right)$ then normal line passes through $(0, b)$ and if we draw tangent at $L\left(ae, \frac{b^2}{a}\right)$ then normal will pass through $(0, -b)$.

Now, equation of normal at $L'\left(ae, -\frac{b^2}{a}\right)$ to the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ is given by $\frac{a^2x}{ae} - \frac{b^2y}{\left(-\frac{b^2}{a}\right)} = a^2 - b^2$

i.e. $\frac{ax}{e} + ya = a^2 - b^2$ passes through $B(0, b)$.

$\Rightarrow 0 + ab = a^2 - b^2 \Rightarrow a^2b^2 = (a^2 - b^2)^2$

$\Rightarrow a^2a^2(1 - e^2) = [a^2 - a^2(1 - e^2)]^2$

(Using $b^2 = a^2(1 - e^2)$)

$\Rightarrow a^4(1 - e^2) = a^4e^4 \Rightarrow e^4 + e^2 - 1 = 0$

$\Rightarrow e^2 = \frac{-1 \pm \sqrt{5}}{2} \quad \left(e^2 = \frac{-1 - \sqrt{5}}{2} \text{ is impossible}\right)$

$\therefore e^2 = \frac{\sqrt{5} - 1}{2} \Rightarrow 2e^2 + 1 = \sqrt{5}$

$\Rightarrow (2e^2 + 1)^2 = 5 \Rightarrow 20(2e^2 + 1)^2 = 100$

22. (256) : If asymptotes of a hyperbola are perpendicular to each other then it is a rectangular hyperbola and eccentricity of a rectangular hyperbola is $\sqrt{2}$. In our problem condition is satisfied

$\therefore e_1 = e_2 = \sqrt{2}$

$\therefore 16\sqrt{2}\, e_1^3\, e_2^4 = 16\sqrt{2}\,(\sqrt{2})^7 = 16(\sqrt{2})^8 = 16 \times 2^4 = 256$

23. (29) : Given equations of asymptotes are $2x + y + 3 = 0$ and $3x + 4y + 5 = 0$

$\therefore$ Combined equation of asymptotes is

$(2x + y + 3)(3x + 4y + 5) = 0$

$\Rightarrow 6x^2 + 4y^2 + 11xy + 19x + 17y + 15 = 0 \quad ...(i)$

Also, we know that the equation of hyperbola differ from that of asymptotes by a constant.

Now, the equation of hyperbola

$6x^2 + 4y^2 + 11xy + 19x + 17y + \lambda = 0 \quad ...(ii)$

passes through $(-1, 1)$

$\therefore 6(-1)^2 + 4(1)^2 + 11(-1)(1) + 19(-1) + 17(1) + \lambda = 0$

$\Rightarrow \lambda = 3$

$\therefore$ (ii) becomes $6x^2 + 4y^2 + 11xy + 19x + 17y + 3 = 0$

Now, equation of conjugate hyperbola is

$2(6x^2 + 4y^2 + 11xy + 19x + 17y + 15)$

$- (6x^2 + 4y^2 + 11xy + 19x + 17y + 3) = 0$

$\Rightarrow 6x^2 + 4y^2 + 11xy + 19x + 17y + 27 = 0$

$= 6x^2 + 4y^2 + 11xy + lx + my + n$

$\Rightarrow l = 19, m = 17, n = 27$

$\therefore l - m + n = 19 - 17 + 27 = 29$

MOCK TEST PAPER 2019
JEE MAIN

Time : 1 hr 15 min. **Series - 7**

The entire syllabus of Mathematics of JEE MAIN is being divided in to eight units, on each unit there will be a Mock Test Paper (MTP) which will be published in the subsequent issues. The syllabus for module break-up is given bellow:

Unit No.	Topic	Syllabus In Details
UNIT NO.7	Co-ordinate Geometry-3D	Direction ratios and direction cosines. Angle between two intersecting lines. Skew lines, the shortest distance between them and its equation. Equation of a line and a plane in different forms, intersection of a line and a plane, coplanar lines.
	Differential calculus	Rolle's and Lagrange's Mean value theorem theorems, Applications of derivatives: Rate of change of quantities, monotonic-increasing and decreasing functions, Maxima and minima of functions of one variable, tangents and normals.
	Integral calculus	Integral as an anti-derivative. Fundamental integrals involving algebraic, trigonometric, exponential and logarithmic functions. Integration by substitution, by parts and by partial fractions. Integration using trigonometric identities. Evaluation of simple integrals of type: $\int\frac{dx}{x^2\pm a^2},\int\frac{dx}{\sqrt{x^2\pm a^2}},\int\frac{dx}{a^2-x^2},\int\frac{dx}{\sqrt{a^2-x^2}},\int\frac{dx}{ax^2+bx+c},\int\frac{dx}{\sqrt{ax^2+bx+c}},\int\frac{(px+q)dx}{ax^2+bx+c}$

1. The projections of a directed line segment on the coordinate axes are12, 4, 3. The direction cosines of the line are

(a) $\frac{12}{13},-\frac{4}{13},\frac{3}{13}$ (b) $-\frac{12}{13},-\frac{4}{13},\frac{3}{13}$

(c) $\frac{12}{13},\frac{4}{13},\frac{3}{13}$ (d) none of these

2. The direction ratio of the line perpendicular to the lines $\frac{x-7}{2}=\frac{y+17}{-3}=\frac{z-6}{1}$ and $\frac{x+5}{1}=\frac{y+3}{2}=\frac{z-4}{-2}$ are proportional to

(a) 4, 5, 7 (b) 4, –5, 7

(c) 4, –5, –7 (d) –4, 5, 7

3. A non-zero vector $\vec{a}$ is parallel to the line of intersection of the plane determined by the vectors $\hat{i},\hat{i}+\hat{j}$ and the plane determined by the vectors $\hat{i}-\hat{j},\hat{i}+\hat{k}$. The angle between $\vec{a}$ and $\hat{i}-2\hat{j}+2\hat{k}$ is

(a) $\frac{\pi}{3}$ (b) $\frac{\pi}{4}$

(c) $\frac{\pi}{6}$ (d) none of these

4. If the plane $\frac{x}{2}+\frac{y}{3}+\frac{z}{4}=1$ cuts the coordinate axes in A, B, C, then the area of ΔABC is

(a) $\sqrt{29}$ sq.units (b) $\sqrt{41}$ sq.units

(c) $\sqrt{61}$ sq.units (d) none of these

5. Equation of the plane that contains the lines $\vec{r}=(\hat{i}+\hat{j})+\lambda(\hat{i}+2\hat{j}-\hat{k})$ and $\vec{r}=(\hat{i}+\hat{j})+\mu(-\hat{i}+\hat{j}-2\hat{k})$ is

(a) $\vec{r}\cdot(2\hat{i}+\hat{j}-3\hat{k})=-4$ (b) $\vec{r}\times(-\hat{i}+\hat{j}+\hat{k})=\vec{0}$

(c) $\vec{r}\cdot(-\hat{i}+\hat{j}+\hat{k})=0$ (d) none of these.

By : Sankar Ghosh, S.G.M.C, Mob. 09831244397.

6. The distance of the point (1, –2, 3) from the plane $x - y + z = 5$ measured parallel to the line $\frac{x}{2} = \frac{y}{3} = \frac{z-1}{-6}$ is

(a) 1 (b) 2
(c) 4 (d) none of these

7. If the lines $\frac{x-1}{2} = \frac{y+1}{3} = \frac{z-1}{4}$ and $\frac{x-3}{1} = \frac{y-k}{2} = \frac{z}{1}$ intersect, then the value of k is

(a) $\frac{2}{3}$ (b) $\frac{9}{2}$ (c) $-\frac{2}{9}$ (d) $-\frac{3}{2}$

8. The length of perpendicular drawn from $P(1, 2, 3)$ to the line $\frac{x-6}{3} = \frac{y-7}{2} = \frac{z-7}{-2}$, is

(a) 4 (b) 5 (c) 6 (d) 7

9. If the angle θ between the line $\frac{x+1}{1} = \frac{y-1}{2} = \frac{z-2}{2}$ and the plane $2x - y + \sqrt{\lambda}z + 4 = 0$ is such that $\sin\theta = \frac{1}{3}$, the value of λ is

(a) $-\frac{4}{3}$ (b) $\frac{3}{4}$ (c) $-\frac{3}{5}$ (d) $\frac{5}{3}$

10. Consider the lines $L_1 : \frac{x+1}{3} = \frac{y+2}{1} = \frac{z+1}{2}$, $L_2 : \frac{x-2}{1} = \frac{y+2}{2} = \frac{z-3}{3}$. The unit vector perpendicular to both L_1 and L_2 is

(a) $\frac{1}{\sqrt{99}}(-\hat{i} + 7\hat{j} + 7\hat{k})$ (b) $\frac{1}{5\sqrt{3}}(-\hat{i} - 7\hat{j} + 5\hat{k})$

(c) $\frac{1}{5\sqrt{3}}(-\hat{i} + 7\hat{j} + 5\hat{k})$ (d) $\frac{1}{\sqrt{99}}(7\hat{i} - 7\hat{j} + \hat{k})$

11. Rolle's theorem hold for the function $f(x) = x^3 + bx^2 + cx$, $1 \le x \le 2$ at the point $\frac{4}{3}$, then values of b and c are

(a) $b = 8, c = -5$ (b) $b = -5, c = 8$
(c) $b = 5, c = -8$ (d) $b = -5, c = -8$

12. Let $f(x)$ satisfy the requirement of Lagrange's Mean Value Theorem in $[0, 2]$. If $f(0) = 0$ and $|f'(x)| \le \frac{1}{2}$ for all $x \in [0, 2]$, then

(a) $f(x) \le 2$
(b) $|f(x)| \le 1$
(c) $f(x) = 2x$
(d) $f(x) = 3$ for at least one x in $[0, 2]$

13. A variable triangle is inscribed in a circle of radius R. If the rate of change of a side is R times the rate of change of the opposite angle, then that angle is

(a) $\frac{\pi}{6}$ (b) $\frac{\pi}{4}$ (c) $\frac{\pi}{3}$ (d) $\frac{\pi}{2}$

14. The period of oscillation T of a pendulum of length l at a place of acceleration due to gravity g is given by $T = 2\pi\sqrt{\frac{l}{g}}$. If the calculated length is 0.992 times the actual length and if the value assumed for g is 1.002 times its actual value, the relative error in the computed value of T is

(a) 0.005 (b) –0.005
(c) 0.003 (d) –0.003

15. Let $f(x) = x^3 + ax^2 + bx + 5\sin^2 x$ be an increasing function on the set R. Then, a and b satisfy

(a) $a^2 - 3b - 15 > 0$ (b) $a^2 - 3b + 15 > 0$
(c) $x^2 - 3b + 15 < 0$ (d) $a > 0$ and $b > 0$

16. Let $f : R \to R$ be a function such that $f(x) = ax + 3\sin x + 4\cos x$. Then, $f(x)$ is invertible if

(a) $a \in (-5, 5)$ (b) $a \in (-\infty, -5)$
(c) $a \in (5, \infty)$ (d) none of these

17. If $f(x) = a\log|x| + bx^2 + x$ has its extremum values at $x = -1$ and $x = 2$, then

(a) $a = 2, b = -1$ (b) $a = 2, b = -\frac{1}{2}$
(c) $a = -2, b = \frac{1}{2}$ (d) none of these

18. A cylindrical container is to be made from certain solid material with the following constraints: It has a fixed inner volume of V mm^3, has a 2mm thick solid wall and is open at the top. The bottom of the container is a solid circular disc of thickness 2 mm and is of radius equal to the outer radius of the container. If the volume of the material used to make the container is minimum when the inner radius of the container is 10 mm, then the value of $\frac{V}{250\pi}$ is

(a) 6 (b) 8 (c) 7 (d) 4

19. If the sum of the squares of the intercepts on the axes cut off by the tangent to the curve $x^{\frac{1}{3}} + y^{\frac{1}{3}} = a^{\frac{1}{3}}$ (with $a > 0$) at $\left(\frac{a}{8}, \frac{a}{8}\right)$ is 2, then a has the value

(a) 1 (b) 2 (c) 4 (d) 8

20. The coordinates of a point on the curve $x = 4t^2 + 3$, $y = 8t^3 - 1$ such that the tangent to the curve at that point is normal to the curve at its other intersection point with the curve, are

(a) $\left(\frac{35}{9}, \pm\frac{16\sqrt{2}}{27} - 1\right)$ (b) $\left(\frac{25}{9}, \pm\frac{11}{7}\right)$

(c) $\left(\frac{35}{9}, \pm\frac{16\sqrt{2}}{27} + 1\right)$ (d) none of these

21. If $\int \frac{\sin^8 x - \cos^8 x}{1 - 2\sin^2 x \cos^2 x}\,dx = a\sin 2x + C$, then $a =$

(a) $-\frac{1}{2}$ (b) $\frac{1}{2}$ (c) -1 (d) 1

22. The value of $\int \sqrt[3]{\frac{\sin^n x}{\cos^{n+6} x}}\,dx =$

(a) $\frac{3}{n+3}\tan^{\frac{n}{3}} x + C$ (b) $\frac{3}{n+3}\tan^{\frac{n}{3}+1} x + C$

(c) $\frac{3}{n+1}\tan^{\frac{n}{3}+1} x + C$ (d) none of these

23. If $\int e^{ax}\cos bx\,dx = \frac{e^{ax}}{29} f(x) + C$, then $f''(x) =$

(a) $29f(x)$ (b) $-29f(x)$

(c) $25f(x)\frac{1}{2\sqrt{2}}$ (d) $-25f(x)$

24. $\int \frac{1}{\sin x - \cos x + \sqrt{2}}\,dx =$

(a) $-\frac{1}{\sqrt{2}}\tan\left(\frac{x}{2} + \frac{\pi}{8}\right) + C$

(b) $\frac{1}{\sqrt{2}}\tan\left(\frac{x}{2} + \frac{\pi}{8}\right) + C$

(c) $\frac{1}{\sqrt{2}}\cot\left(\frac{x}{2} + \frac{\pi}{8}\right) + C$

(d) $-\frac{1}{\sqrt{2}}\cot\left(\frac{x}{2} + \frac{\pi}{8}\right) + C$.

25. $\int \frac{1}{(2x-7)\sqrt{x^2 - 7x + 12}}\,dx =$

(a) $2\sec^{-1}(2x - 7) + C$ (b) $\sec^{-1}(2x - 7) + C$

(c) $\frac{1}{2}\sec^{-1}(2x-7) + C$ (d) none of these.

26. Let $f(x)$ be a quadratic function such that $f(0) = 1$ and $\int \frac{f(x)}{x^2(x+1)^3}\,dx$ is a rational function, then the value of $f'(0)$ is

(a) 0 (b) 2 (c) 3 (d) 5

27. $\int x^2 \frac{(x\sec^2 x + \tan x)}{(x\tan x + 1)^2}\,dx =$

(a) $\frac{-x}{(x\tan x + 1)} + 2\ln|x\sin x + \cos x| + C$

(b) $\frac{-x^2}{(x\tan x + 1)} + 2\ln|x\sin x + \cos x| + C$

(c) $\frac{-x^2}{(x\tan x + 1)} + 2\ln|x\sin x - \cos x| + C$

(d) none of these

28. $\int \frac{\tan x}{\sqrt{\sin^4 x + \cos^4 x}}\,dx =$

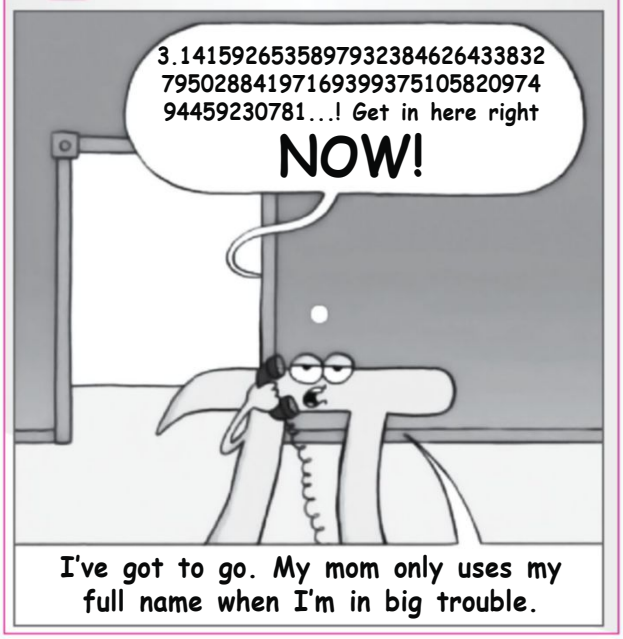
(a) $\log_e(\tan^2 x + \sqrt{1 + \tan^4 x}) + C$

(b) $\frac{1}{2}\log_e(\tan^2 x + \sqrt{1 + \tan^4 x}) + C$

(c) $\frac{1}{4}\log_e(\tan^2 x + \sqrt{1 + \tan^4 x}) + C$

(d) none of these

COMIC CAPSULE



29. $\int x\log\left(1+\frac{1}{x}\right)dx = f(x)\cdot\log(x+1) + g(x)\log x^2 + Lx + C$, then

(a) $f(x)=\frac{x^2}{2}$ (b) $g(x) = \log_e x$

(c) $L = 1$ (d) $L=\frac{1}{2}$

30. $\int\frac{\sin x+8\cos x}{4\sin x+6\cos x}dx =$

(a) $x+\frac{1}{2}\log|4\sin x+6\cos x|+C$

(b) $2x + \log|2\sin x + 3\cos x| + C$

(c) $x + 2\log|2\sin x + 3\cos x| + C$

(d) $\frac{1}{2}\log|4\sin x+6\cos x|+C$.

SOLUTIONS

1. (c) : Let the directed line segment be r and its direction cosines be l, m, n. Then, its projections on the coordinate axes are lr, mr, nr.

$\therefore$ $lr = 12$, $mr = 4$, $nr = 3$

$\Rightarrow$ $l^2r^2 + m^2r^2 + n^2r^2 = 144 + 16 + 9$

$\Rightarrow$ $r^2(l^2 + m^2 + n^2) = 169$

$\Rightarrow$ $r^2 = 169 \Rightarrow r = 13$ $[\because\ l^2 + m^2 + n^2 = 1]$

$\therefore$ $l=\frac{12}{13},\ m=\frac{4}{13},\ n=\frac{3}{13}$

Hence, the direction cosines of the line are $\frac{12}{13}, \frac{4}{13}, \frac{3}{13}$.

2. (a) : Let a, b, c are the direction ratios of the line perpendicular to the lines

$$\frac{x-7}{2}=\frac{y+17}{-3}=\frac{z-6}{1} \text{ and } \frac{x+5}{1}=\frac{y+3}{2}=\frac{z-4}{-2}$$

$\therefore$ $2a - 3b + c = 0$ and $a + 2b - 2c = 0$

$\Rightarrow$ $\frac{a}{4}=\frac{b}{5}=\frac{c}{7} \Rightarrow a:b:c=4:5:7$

3. (b) : Let $\vec{n}_1$ = Normal vector to the plane determined by $\hat{i}$ and $\hat{i}+\hat{j}$

$\therefore$ $\vec{n}_1=\hat{i}\times(\hat{i}+\hat{j})=\hat{k}$

Let $\vec{n}_2$ = Normal vector to the plane determined by $\hat{i}-\hat{j}$ and $\hat{i}+\hat{k}$

$\therefore$ $\vec{n}_2=(\hat{i}-\hat{j})\times(\hat{i}+\hat{k})=-\hat{i}-\hat{j}+\hat{k}$

According to the problem, $\vec{a}$ is perpendicular to the normals to the two planes determined by the given pair of vectors.

Since $\vec{a}$ is perpendicular to $\vec{n}_1$ and $\vec{n}_2$, therefore

$\vec{a}=\lambda(\vec{n}_1\times\vec{n}_2)=\lambda|\hat{k}\times(-\hat{i}-\hat{j}+\hat{k})|=\lambda(\hat{i}-\hat{j})$

Let θ be the angle between $\vec{a}$ and $(\hat{i}-2\hat{j}+2\hat{k})$. Then,

$$\cos\theta=\frac{\lambda(1+2+0)}{\lambda\sqrt{2}\sqrt{1+4+4}}=\frac{1}{\sqrt{2}}\Rightarrow\theta=\frac{\pi}{4}$$

4. (c) : The given plane cuts the coordinate axes in $A(2, 0, 0)$, $B(0, 3, 0)$ and $C(0, 0, 4)$.

$\therefore$ Area of $\Delta ABC=\frac{1}{2}AB\times AC\times\sin\angle BAC$ ...(i)

Now, $AB=\sqrt{(0-2)^2+(3-0)^2+(0-0)^2}=\sqrt{4+9}=\sqrt{13}$,

$AC=\sqrt{(0-2)^2+(0-0)^2+(4-0)^2}=\sqrt{4+16}=\sqrt{20}$

And $\cos\angle BAC=\frac{\overrightarrow{AB}\cdot\overrightarrow{AC}}{|\overrightarrow{AB}||\overrightarrow{AC}|}=\frac{(-2\hat{i}+3\hat{j})\cdot(-2\hat{i}+4\hat{k})}{\sqrt{4+9}\sqrt{4+16}}$

$=\frac{4+0+0}{\sqrt{13}\sqrt{20}}=\frac{4}{\sqrt{13}\sqrt{20}}=\frac{2}{\sqrt{65}}$

$\therefore$ $\sin\angle BAC=\sqrt{1-\left(\frac{2}{\sqrt{65}}\right)^2}=\sqrt{\frac{61}{65}}$

Now (i) becomes area of ΔABC

$=\frac{1}{2}\times\sqrt{13}\times\sqrt{20}\times\sqrt{\frac{61}{65}}=\sqrt{61}$ sq.units

5. (c) : The lines are parallel to the vectors $\vec{b}_1=\hat{i}+2\hat{j}-\hat{k}$ and $\vec{b}_2=-\hat{i}+\hat{j}-2\hat{k}$. Therefore, the plane is normal to the vector $\vec{n}=\vec{b}_1\times\vec{b}_2=\begin{vmatrix}\hat{i}&\hat{j}&\hat{k}\\1&2&-1\\-1&1&-2\end{vmatrix}=-3\hat{i}+3\hat{j}+3\hat{k}$.

The required plane passes through $\vec{a}=(\hat{i}+\hat{j})$ and is normal to the vector $\vec{n}$.

$\therefore$ Equation of plane is $\vec{r}\cdot\vec{n}=\vec{a}\cdot\vec{n}$

$\Rightarrow$ $\vec{r}\cdot(-3\hat{i}+3\hat{j}+3\hat{k})=(\hat{i}+\hat{j})\cdot(-3\hat{i}+3\hat{j}+3\hat{k})=-3+3=0$

$\Rightarrow$ $\vec{r}\cdot(-\hat{i}+\hat{j}+\hat{k})=0$

6. (a) : The equation of the line passing through $P(1, -2, 3)$ and parallel to the given line is

$$\frac{x-1}{2}=\frac{y+2}{3}=\frac{z-3}{-6} \quad ...(i)$$

Let the line (i) meets the plane $x - y + z = 5$ at the point Q given by

$$\frac{x-1}{2}=\frac{y+2}{3}=\frac{z-3}{-6}=\lambda \text{ i.e., } (2\lambda+1, 3\lambda-2, -6\lambda+3)$$

This lies on $x - y + z = 5$. Therefore,

$$2\lambda+1-(3\lambda-2)+(-6\lambda+3)=5 \Rightarrow -7\lambda=-1 \Rightarrow \lambda=\frac{1}{7}$$

Thus, the coordinates of Q are

$$\left(\frac{2}{7}+1, \frac{3}{7}-2, -\frac{6}{7}+3\right)=\left(\frac{9}{7}, -\frac{11}{7}, \frac{15}{7}\right).$$

Hence, the required distance,

$$PQ=\sqrt{\frac{4}{49}+\frac{9}{49}+\frac{36}{49}}=1$$

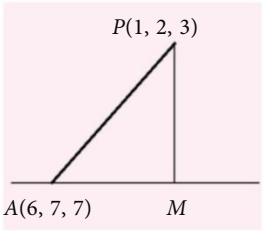
7. (b): The lines $\frac{x-1}{2}=\frac{y+1}{3}=\frac{z-1}{4}$ and $\frac{x-3}{1}=\frac{y-k}{2}=\frac{z}{1}$ intersects when

$$\begin{vmatrix} 3-1 & k+1 & 0-1 \\ 2 & 3 & 4 \\ 1 & 2 & 1 \end{vmatrix}=0$$

$$\Rightarrow \quad 2(3-8)-(k+1)(2-4)-(4-3)=0 \Rightarrow k=\frac{9}{2}$$

8. (d): The line $\frac{x-6}{3}=\frac{y-7}{2}=\frac{z-7}{-2}$ passes through $A(6, 7, 7)$ and parallel to the vector $\vec{b}=3\hat{i}+2\hat{j}-2\hat{k}$.

Let M be the foot of the perpendicular from $P(1, 2, 3)$ on the given line.



We have, $\overrightarrow{AP}=-5\hat{i}-5\hat{j}-4\hat{k}$

$$\Rightarrow |\overrightarrow{AP}|=\sqrt{25+25+16}=\sqrt{66}$$

Here, AM = Projection of $\overrightarrow{AP}$ on $\vec{b}$

$$=\left|\frac{\overrightarrow{AP}\cdot\vec{b}}{|\vec{b}|}\right|=\frac{(5\hat{i}+5\hat{j}+4\hat{k})\cdot(3\hat{i}+2\hat{j}-2\hat{k})}{|3\hat{i}+2\hat{j}-2\hat{k}|}$$

$$=\frac{15+10-8}{\sqrt{9+4+4}}=\sqrt{17}$$

$$\therefore \quad PM=\sqrt{AP^2-AM^2} \Rightarrow PM=\sqrt{66-17}=7$$

9. (d): The given line is parallel to the vector $\vec{b}=\hat{i}+2\hat{j}+2\hat{k}$ and the given plane is normal to the vector $\vec{n}=2\hat{i}-\hat{j}+\sqrt{\lambda}\,\hat{k}$

$$\therefore \quad \sin\theta=\frac{\vec{b}\cdot\vec{n}}{|\vec{b}||\vec{n}|} \Rightarrow \frac{1}{3}=\frac{2-2+2\sqrt{\lambda}}{3\times\sqrt{\lambda+5}} \Rightarrow \lambda=\frac{5}{3}$$

10. (b): Lines L_1 and L_2 are parallel to the vectors $\vec{b}_1=3\hat{i}+\hat{j}+2\hat{k}$ and $\vec{b}_2=\hat{i}+2\hat{j}+3\hat{k}$ respectively. Therefore the unit vector perpendicular to both L_1 and L_2 is

$$\hat{n}=\frac{\vec{b}_1\times\vec{b}_2}{|\vec{b}_1\times\vec{b}_2|} \qquad ...(i)$$

Now, $\vec{b}_1\times\vec{b}_2=\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 1 & 2 \\ 1 & 2 & 3 \end{vmatrix}=-\hat{i}-7\hat{j}+5\hat{k}$

From (i), we get $\hat{n}=\frac{1}{5\sqrt{3}}(-\hat{i}-7\hat{j}+5\hat{k})$.

SAMURAI SUDOKU

Samurai Sudoku puzzle consists of five overlapping sudoku grids. The standard sudoku rules apply to each 9 × 9 grid. Place digits from 1 to 9 in each empty cell. Every row, every column and every 3 × 3 box should contain one of each digit.

The puzzle has a unique solution.

			6									5							
		5		1								4		6					
		8			7	3	1					8			2	9	7		
		9		4				3				3		8				2	
	5		8		9		4					7		1		5		3	
4				3		9						2				7		6	
	3	1	4			2			8				5	7	2			8	
				7		5		8		2						3		5	
					6			1			5	8	2				6		
								5		6				4					
							9		4		7		6						
						6				5		9							
			8				5	4	2			1			6				
		9		2						7		4		2		1			
		6			3	9	1				8			6			9	2	8
		4		7				9						1		5			2
	7		2		8		4						4		7		1		9
5				4		6						8				4		3	
	9	7	6			4							1	9	2			6	
				8		2										7		9	
					5												4		

Readers can send their responses at editor@mtg.in or post us with complete address. Winners' name with their valuable feedback will be published in next issue.

11. (b) : Since, $f(x) = x^3 + bx^2 + cx$ satisfies the rolle's theorem on $[1, 2]$.

$\therefore \quad f(1) = f(2)$ and $f'\left(\frac{4}{3}\right) = 0$, where $f'(x) = 3x^2 + 2bx + c$

$\Rightarrow \quad 1 + b + c = 8 + 4b + 2c$ and

$3\left(\frac{4}{3}\right)^2 + 2b\left(\frac{4}{3}\right) + c = 0$

$\Rightarrow \quad 3b + c + 7 = 0$ and $8b + 3c + 16 = 0 \Rightarrow b = -5, c = 8$

12. (b) : We are given that $f(x)$ satisfies all the conditions of Lagrange's mean value theorem in $[0, 2]$. So it also satisfies in$[0, x]$.Consequently, there exist $c \in (0, x)$,where $x \in (0, 2)$ such that

$$f'(c) = \frac{f(x) - f(0)}{x - 0} \Rightarrow f'(c) = \frac{f(x)}{x} \quad [\because f(0) = 0]$$

$$\Rightarrow \left|\frac{f(x)}{x}\right| = |f'(c)| \le \frac{1}{2} \quad \left[\because |f'(x)| \le \frac{1}{2}\right]$$

$$\Rightarrow |f(x)| \le \frac{|x|}{2} \Rightarrow |f(x)| \le \frac{x}{2}$$

$\Rightarrow |f(x)| \le 1 \quad [\because \quad x \in (0, 2) \quad \therefore \quad |x| < 2]$

13. (c) : Let the side $BC = a$ and A be the opposite angle. Then,

$$R = \frac{a}{2\sin A} \Rightarrow a = 2R\sin A$$

$$\Rightarrow \frac{da}{dt} = 2R\cos A \frac{dA}{dt}$$

$$\Rightarrow R\frac{dA}{dt} = 2R\cos A \cdot \frac{dA}{dt} \quad \left[\because \frac{da}{dt} = R\frac{dA}{dt} \text{ (given)}\right]$$

$$\Rightarrow \cos A = \frac{1}{2} \Rightarrow A = \frac{\pi}{3}$$

14. (b) : We have, $l + \Delta l = 0.992l$ and $g + \Delta g = 1.002g$

$\Rightarrow \Delta l = -0.008l$ and $\Delta g = 0.002g$

$$\Rightarrow \frac{\Delta l}{l} = -0.008 \text{ and } \frac{\Delta g}{g} = 0.002$$

Now, $T = 2\pi\sqrt{\frac{l}{g}}$

$$\Rightarrow \log T = \log 2\pi + \frac{1}{2}(\log l - \log g)$$

$$\Rightarrow d(\log T) = d(\log 2\pi) + \frac{1}{2}d(\log l - \log g)$$

$$\Rightarrow \frac{dT}{T} = \frac{1}{2}\left(\frac{dl}{l} - \frac{dg}{g}\right) \Rightarrow \frac{\Delta T}{T} = \frac{1}{2}\left(\frac{\Delta l}{l} - \frac{\Delta g}{g}\right)$$

$[\because dT \cong \Delta T, dl \cong \Delta l \text{ and } dg \cong \Delta g]$

$$\Rightarrow \frac{\Delta T}{T} = \frac{1}{2}(-0.008 - 0.002) = -0.005$$

15. (c) : We have, $f(x) = x^3 + ax^2 + bx + 5\sin^2 x$

$\therefore \quad f'(x) = 3x^2 + 2ax + b + 5\sin 2x$

Since $f(x)$ is an increasing function on R, therefore $f'(x) > 0 \ \forall \ x \in R$.

$\Rightarrow \quad 3x^2 + 2ax + (b - 5) > 0 \ \forall \ x \in R$

$\Rightarrow \quad (2a)^2 - 4 \times 3 \times (b - 5) < 0 \Rightarrow a^2 - 3b + 15 < 0$

16. (d) : We have, $f(x) = ax + 3\sin x + 4\cos x$

$\therefore \quad f'(x) = a + 3\cos x - 4\sin x$

$f(x)$ will be invertible, when $f'(x) > 0 \ \forall \ x$

or $f'(x) < 0 \ \forall \ x$

$\Rightarrow \quad a + 3\cos x - 4\sin x > 0 \ \forall \ x$

or $\quad a + 3\cos x - 4\sin x < 0 \ \forall \ x$

$\Rightarrow \quad a - 5 > 0$ or $a + 5 < 0 \quad [\because \ -5 \le 3\cos x - 4\sin x \le 5]$

$\Rightarrow \quad a > 5$ or $a < -5 \Rightarrow \quad a \in (-\infty, -5) \cup (5, \infty)$

17. (b) : We have, $f(x) = a\log|x| + bx^2 + x$

$$\therefore \quad f'(x) = \frac{a}{x} + 2bx + 1$$

$\therefore \quad f(x)$ attains its maximum values at $x = -1, 2$

$\therefore \quad f'(-1) = 0$ and $f'(2) = 0$.

$$\Rightarrow \quad -a - 2b + 1 = 0 \text{ and } \frac{a}{2} + 4b + 1 = 0$$

$$\Rightarrow \quad a = 2 \text{ and } b = -\frac{1}{2}.$$

18. (d) : Let the inner radius of the container be r mm and height be h mm. Then, $V = \pi r^2 h$

Let M be the volume of material used in making container. Then,

$M = \pi(r + 2)^2(h + 2) - \pi r^2 h$

$$\Rightarrow \quad M = \pi(r+2)^2\left(\frac{V}{\pi r^2} + 2\right) - V$$

$$\Rightarrow \quad M = V\left(\frac{4}{r} + \frac{4}{r^2}\right) + 2\pi(r+2)^2$$

$$\Rightarrow \quad \frac{dM}{dr} = V\left(-\frac{4}{r^2} - \frac{8}{r^3}\right) + 4\pi(r+2)$$

It is given that M is minimum when $r = 10$ mm.

$$\therefore \quad \left(\frac{dM}{dr}\right) = 0 \Rightarrow V\left(-\frac{4}{100} - \frac{8}{1000}\right) + 48\pi = 0$$

$$\Rightarrow \quad V = 1000\pi \Rightarrow \frac{V}{250\pi} = 4.$$

19. (c) : We have, $x^{1/3} + y^{1/3} = a^{1/3}$ ...(i)

Differentiating (i) w.r.t. x, we have

$$\frac{1}{3}x^{-\frac{2}{3}} + \frac{1}{3}y^{-\frac{2}{3}}\frac{dy}{dx} = 0 \Rightarrow \frac{dy}{dx} = -\frac{x^{-\frac{2}{3}}}{y^{-\frac{2}{3}}} = -\frac{y^{\frac{2}{3}}}{x^{\frac{2}{3}}}$$

$$\Rightarrow \quad \left(\frac{dy}{dx}\right)_{(a/8,a/8)} = -1$$

The equation of the tangent at $(a/8, a/8)$ is given by

$$y-\frac{a}{8}=-1\left(x-\frac{a}{8}\right)\Rightarrow x+y-\frac{a}{4}=0\Rightarrow \frac{x}{\frac{a}{4}}+\frac{y}{\frac{a}{4}}=1$$

The intercepts on the coordinate axes are each equal to $\frac{a}{4}$.

$$\therefore \quad \left(\frac{a}{4}\right)^2+\left(\frac{a}{4}\right)^2=2\Rightarrow a=4$$

20. (a) : We have,

$$x=4t^2+3 \text{ and } y=8t^3-1 \;\Rightarrow\; \frac{dy}{dx}=3t$$

The equation of the tangent at any point 't' is

$$y - 8t^3 + 1 = 3t(x - 4t^2 - 3)$$

Let its meets the curve again at the point 't_1'. Then,

$$8t_1^3-1-8t^3+1=3t(4t_1^2+3-4t^2-3)$$

$$\Rightarrow \quad 8(t_1^3-t^3)=12t(t_1^2-t^2)$$

$$\Rightarrow \quad 4(t_1-t)(2t_1^2+2t_1t+2t^2-3tt_1-3t^2)=0$$

$$\Rightarrow \quad 4(t_1-t)(2t_1^2-tt_1-t^2)=0$$

$$\Rightarrow \quad 4(t_1-t)^2(2t_1+t)=0\Rightarrow t_1=-\frac{t}{2}$$

The slope of the normal at t_1 is $-\frac{1}{3t_1}$.

$$\therefore \quad 3t=-\frac{1}{3t_1}\Rightarrow 9tt_1=-1\Rightarrow -9t^2=-2\left[\because t_1=-\frac{1}{2}t\right]$$

$$\Rightarrow \quad t=\pm\frac{\sqrt{2}}{3}$$

Hence, the required points are $\left(\frac{35}{9},\pm\frac{16\sqrt{2}}{27}-1\right)$.

21. (a) : Let $I=\int\frac{\sin^8 x-\cos^8 x}{1-2\sin^2 x\cos^2 x}\,dx$

$$=\int\frac{(\sin^4 x+\cos^4 x)(\sin^4 x-\cos^4 x)}{1-2\sin^2 x\cos^2 x}\,dx$$

$$=\int\frac{(\sin^4 x+\cos^4 x)(\sin^2 x+\cos^2 x)(\sin^2 x-\cos^2 x)}{(\sin^2 x+\cos^2 x)^2-2\sin^2 x\cos^2 x}\,dx$$

$$=\int\frac{(\sin^4 x+\cos^4 x)(\sin^2 x+\cos^2 x)(\sin^2 x-\cos^2 x)}{(\sin^4 x+\cos^4 x)}\,dx$$

$$=\int(\sin^2 x-\cos^2 x)\,dx=-\int\cos 2x\,dx$$

$$=-\frac{1}{2}\sin 2x+C=a\sin 2x+C \;\Rightarrow\; a=-\frac{1}{2}$$

22. (b) : Let $I=\int\sqrt[3]{\frac{\sin^n x}{\cos^{n+6} x}}\,dx$

$$=\int\sqrt[3]{\left(\frac{\sin x}{\cos x}\right)^n}\times\frac{1}{\cos^2 x}\,dx=\int\tan^{\frac{n}{3}}x\sec^2 x\,dx$$

$$=\int\tan^{\frac{n}{3}}x\,d(\tan x)=\frac{(\tan x)^{\frac{n}{3}+1}}{\frac{n}{3}+1}+C=\frac{3}{n+3}\tan^{\frac{n}{3}+1}x+C$$

23. (d): We know that

$$\int e^{ax}\cos bx\,dx=\frac{e^{ax}}{a^2+b^2}(a\cos bx+b\sin bx)+C$$

$\Rightarrow$ $a = 2$, $b = 5$ and $f(x) = a\cos bx + b\sin bx$

$\Rightarrow$ $f'(x) = -ab\sin bx + b^2\cos bx$

$\Rightarrow$ $f''(x) = -ab^2\cos bx - b^3\sin bx$

$= -b^2(a\cos bx + b\sin bx) = -b^2 f(x)$

$\Rightarrow$ $f''(x) = -25f(x)$

Note : If we consider $a = 5$, $b = 2$ we get $f''(x) = -4f(x)$ which is not given in the options.

24. (d) : Let $I=\int\frac{dx}{\sin x-\cos x+\sqrt{2}}$

$$=\frac{1}{\sqrt{2}}\int\frac{dx}{\left(\frac{1}{\sqrt{2}}\sin x-\frac{1}{\sqrt{2}}\cos x\right)+1}$$

$$=\frac{1}{\sqrt{2}}\int\frac{dx}{1-\cos\left(x+\frac{\pi}{4}\right)}=\frac{1}{2\sqrt{2}}\int\operatorname{cosec}^2\left(\frac{x}{2}+\frac{\pi}{8}\right)dx$$

$$=\frac{1}{\sqrt{2}}\int\operatorname{cosec}^2\left(\frac{x}{2}+\frac{\pi}{8}\right)d\left(\frac{x}{2}+\frac{\pi}{8}\right)$$

$$=-\frac{1}{\sqrt{2}}\cot\left(\frac{x}{2}+\frac{\pi}{8}\right)+C$$

25. (b) : $\int\frac{dx}{(2x-7)\sqrt{x^2-7x+12}}$

$$=\int\frac{2dx}{(2x-7)\sqrt{4x^2-28x+48}}$$

$$=\int\frac{d(2x-7)}{(2x-7)\sqrt{(2x-7)^2-1}}=\sec^{-1}(2x-7)+C$$

26. (c) : Let $g(x)=\int\frac{f(x)}{x^2(x+1)^3}\,dx.$

Since $f(x)$ is a quadratic function, so let

$$\frac{f(x)}{x^2(x+1)^3}=\frac{A}{x}+\frac{B}{x^2}+\frac{C}{x+1}+\frac{D}{(x+1)^2}+\frac{E}{(x+1)^3}$$

$$\Rightarrow\ g(x)=\int\left\{\frac{A}{x}+\frac{B}{x^2}+\frac{C}{x+1}+\frac{D}{(x+1)^2}+\frac{E}{(x+1)^3}\right\}dx \quad \ldots (i)$$

$$\Rightarrow\ g(x)=A\ln x-\frac{B}{x}+C\ln(x+1)-\frac{D}{(x+1)}-\frac{E}{2(x+1)^2}+K$$

Given that $g(x)$ is a rational function, therefore $A = C = 0$

Putting $A = C = 0$ in (i) we get

$$g(x)=\int\left\{\frac{B(x+1)^3+Dx^2(x+1)+Ex^2}{x^2(x+1)^3}\right\}dx$$

$$\therefore\quad f(x)=B(x+1)^3+Dx^2(x+1)+Ex^2 \qquad \left[\because\ g(x)=\int\frac{f(x)}{x^2(x+1)^3}\,dx\right]$$

$\Rightarrow\ f(x) = (B + D)x^3 + (3B + D + E)x^2 + 3Bx + B$

But, $f(x)$ is a quadratic function such that $f(0) = 1$.

$\therefore\ B + D = 0$ and $B = 1\ \Rightarrow\ B = 1, D = -1$

$\therefore\ f(x) = (2 + E)x^2 + 3x + 1$

$\Rightarrow\ f'(x) = 2(2 + E)x + 3\ \Rightarrow\ f'(0) = 3$

27. (b) : Let $I=\int x^2\frac{(x\sec^2 x+\tan x)}{(x\tan x+1)^2}dx$

$$=\int\frac{x^2}{(x\tan x+1)^2}d(x\tan x+1)$$

$$=x^2\int\frac{d(x\tan x+1)}{(x\tan x+1)^2}-\int\left\{\frac{d(x^2)}{dx}\int\frac{d(x\tan x+1)}{(x\tan x+1)^2}\right\}dx$$

$$=-x^2\times\frac{1}{(x\tan x+1)}+2\int x\times\frac{1}{(x\tan x+1)}dx$$

$$=-\frac{x^2}{(x\tan x+1)}+2\int\frac{x\cos x}{x\sin x+\cos x}dx$$

$$=-\frac{x^2}{(x\tan x+1)}+2\ln|x\sin x+\cos x|+C$$

28. (b) : Let $I=\int\frac{\tan x}{\sqrt{\sin^4 x+\cos^4 x}}dx=\int\frac{\tan x\sec^2 x}{\sqrt{\tan^4 x+1}}dx$

$$=\frac{1}{2}\int\frac{d(\tan^2 x)}{\sqrt{(\tan^2 x)^2+1}}=\frac{1}{2}\log_e\left|\tan^2 x+\sqrt{1+\tan^4 x}\right|+C$$

29. (d) : Let $I=\int x\log\left(1+\frac{1}{x}\right)dx$

$$=\int x\log(1+x)\,dx-\int x\log x\,dx$$

$$=\left\{\frac{x^2}{2}\log(1+x)-\int\frac{x^2}{2(1+x)}dx\right\}-\left\{\frac{x^2}{2}\log x-\int\frac{1}{x}\times\frac{x^2}{2}dx\right\}$$

$$=\frac{x^2}{2}\log(1+x)-\frac{1}{2}\int\frac{x^2-1+1}{(1+x)}dx-\frac{x^2}{2}\log x+\frac{1}{4}x^2+C$$

$$=\frac{x^2}{2}\log(1+x)-\frac{1}{2}\left\{\frac{x^2}{2}-x+\log(1+x)\right\}-\frac{x^2}{2}\log x+\frac{1}{4}x^2+C$$

$$=\frac{x^2}{2}\log(1+x)+\frac{x}{2}-\frac{1}{2}\log(1+x)-\frac{x^2}{2}\log x+C$$

$$=\left(\frac{x^2-1}{2}\right)\log(1+x)-\frac{x^2}{4}\log x^2+\frac{x}{2}+C$$

Therefore, $f(x)=\frac{x^2-1}{2}$, $g(x)=-\frac{x^2}{4}$ and $L=\frac{1}{2}$

30. (a) : Let $I=\int\frac{\sin x+8\cos x}{4\sin x+6\cos x}dx$

Let $\sin x + 8\cos x = l(4\sin x + 6\cos x) + m(4\cos x - 6\sin x)$

Then, $1 = 4l - 6m$ and $8 = 6l + 4m$

$\Rightarrow\ l = 1, m = 1/2$

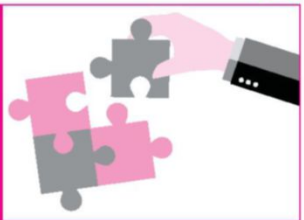
$$\therefore\ I=\int\frac{l(4\sin x+6\cos x)+m(4\cos x-6\sin x)}{4\sin x+6\cos x}dx$$

$$=\int\frac{(4\sin x+6\cos x)+\frac{1}{2}(4\cos x-6\sin x)}{4\sin x+6\cos x}dx$$

$$=x+\frac{1}{2}\log|4\sin x+6\cos x|+C$$

PUZZLE CORNER

SOLUTION - JANUARY 2019



3 3	1− 5	6× 1	4− 6	2	1− 4
1− 4	6	2	3+ 1	3 3	5
5	1 1	3	2	18+ 4	6
15× 1	3	5	20× 4	6	2
3+ 2	32× 4	6 6	5	15× 1	4+ 3
6	2	4	3	5	1

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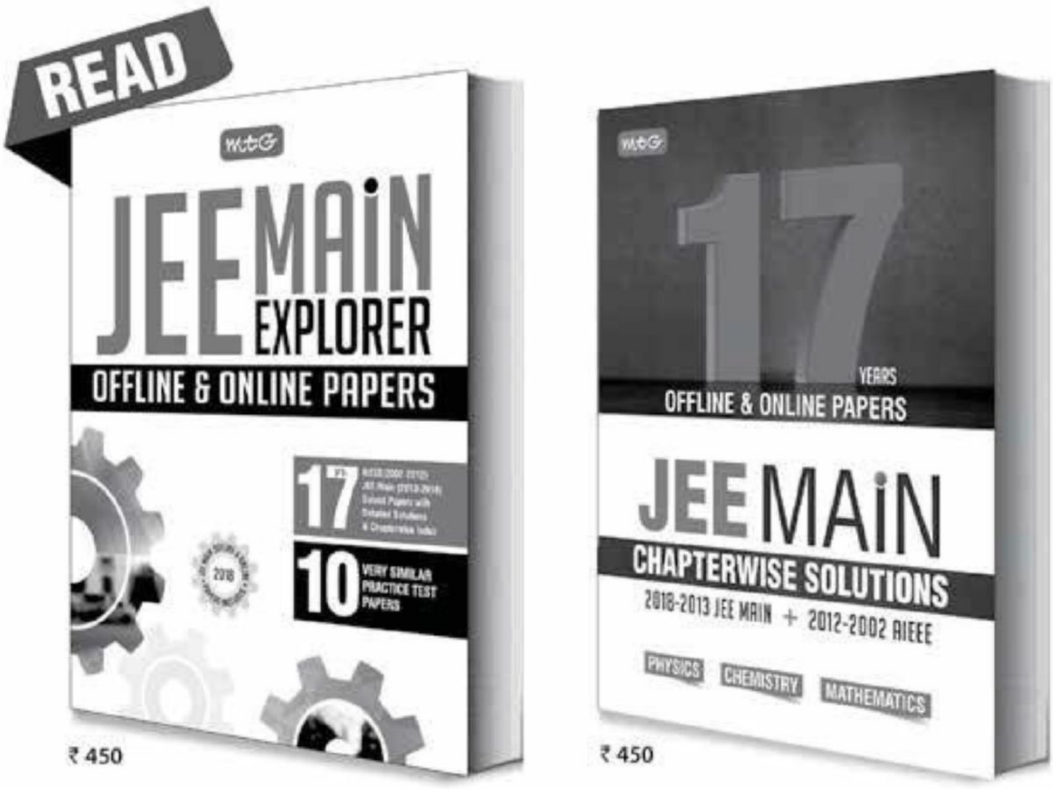

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Exam Between 6th to 20th April 2019

* ALOK KUMAR

1. In a triangle ABC, if $A=\frac{\pi}{7}, B=\frac{2\pi}{7}, C=\frac{4\pi}{7}$, then $a^2+b^2+c^2$ is (R is circumradius of ΔABC)
(a) $4R^2$ (b) $6R^2$ (c) $7R^2$ (d) $8R^2$

2. Range of $f(x) = \sin^6 x + \cos^6 x$ is
(a) $[0, 1]$ (b) $[0, \sqrt{2}]$
(c) $\left[\frac{1}{\sqrt{2}}, \frac{3}{4}\right]$ (d) $\left[\frac{1}{4}, 1\right]$

3. Let $f(x) = x^2 + 5x + 6$, then the number of real roots of $(f(x))^2 + 5f(x) + 6 - x = 0$ is
(a) 1 (b) 2 (c) 3 (d) 0

4. Let $f(x)$ be a function such that $f(x) = x - [x]$, where $[x]$is the greatest integer less than or equal to x. Then the number of solutions of the equation $f(x)+f\left(\frac{1}{x}\right)=1$ is (are)
(a) 0 (b) 1 (c) 2 (d) infinite

5. If z_1, z_2, z_3 and z_4 be the consecutive vertices of a square, then $z_1^2+z_2^2+z_3^2+z_4^2$ equals
(a) $z_1z_2 + z_2z_3 + z_3z_4 + z_4z_1$
(b) $z_1z_2 + z_1z_3 + z_1z_4 + z_2z_3 + z_2z_4 + z_3z_4$
(c) 0 (d) none of these

6. If $|z - 3i| = 3$, (where $i=\sqrt{-1}$) and arg $z \in (0, \pi/2)$, then cot (arg (z)) $-\frac{6}{z}$ is equal to
(a) 0 (b) $-i$
(c) i (d) none of these

7. If the arithmetic mean of two positive numbers a and $b(a > b)$ is twice their G.M., then $a : b$ is
(a) $6+\sqrt{7}:6-\sqrt{7}$ (b) $2+\sqrt{3}:2-\sqrt{3}$
(c) $5+\sqrt{6}:5-\sqrt{6}$ (d) none of these

8. For each positive integer n, let
$S_n=\frac{3}{1\cdot2\cdot4}+\frac{4}{2\cdot3\cdot5}+\frac{5}{3.4.6}+\ldots+\frac{n+2}{n(n+1)(n+3)}$.
Then $\lim\limits_{n\to\infty} S_n$ equals
(a) $\frac{29}{6}$ (b) $\frac{29}{36}$ (c) 0 (d) $\frac{29}{18}$

9. $\lim\limits_{x\to\infty} \sqrt[3]{x}\left(\sqrt[3]{(x+1)^2}-\sqrt[3]{(x-1)^2}\right)=$
(a) $\frac{1}{3}$ (b) $\frac{2}{3}$ (c) 1 (d) $\frac{4}{3}$

10. The value of $\lim\limits_{x\to\infty}\left(\sqrt{x+\sqrt{x+\sqrt{x}}}-\sqrt{x}\right)$ is
(a) 0 (b) $\frac{1}{2}$ (c) $\frac{1}{4}$ (d) 1

11. The range of $\sin^{-1}x - \cos^{-1}x$ is
(a) $\left[\frac{-3\pi}{2}, \frac{\pi}{2}\right]$ (b) $\left[\frac{-5\pi}{2}, \frac{\pi}{3}\right]$
(c) $\left[\frac{-3\pi}{2}, \pi\right]$ (d) $\left[0, \frac{\pi}{2}\right]$

12. If a real valued function f satisfies $f(10 + x) = f(10 - x)$ and $f(20 - x) = -f(20 + x)$, for all $x \in R$ then which of the following statements is true?
(a) f is an even function (b) f is an odd function
(c) f is a constant function
(d) f is a non-periodic function

13. Number of pairs of positive integers (p, q) whose L.C.M. (Least common multiple) is 8100, is "K". Then number of ways of expressing K as a product of two co-prime numbers is
(a) 2 (b) 6 (c) 4 (d) 8

14. If $A=\begin{bmatrix} i & -i \\ -i & i \end{bmatrix}$ and $B=\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$, then $A^8 =$
(a) $4B$ (b) $128B$ (c) $-128B$ (d) $-64\,B$

15. In ΔABC, if $A - B = 120°$ and $R = 8r$, then the value of $\frac{1+\cos C}{1-\cos C}$ equals (All symbols used have their usual meaning in a triangle)
(a) 12 (b) 15 (c) 21 (d) 31

*Alok Kumar, a B Tech from IIT Kanpur and INMO 4th ranker of his time, has been training IIT and Olympiad aspirants for close to two decades now. His students have bagged AIR 1 in IIT JEE and also won medals for the country at IMO. He has also taught at Maths Olympiad programme at Cornell University, USA and UT, Dallas. He has been regularly proposing problems in international Mathematics journals.

16. The domain of the derivative of the function $f(x)=\begin{cases}\tan^{-1}x, & |x|\le 1\\ \frac{1}{2}(|x|-1), & |x|>1\end{cases}$ is

(a) $R-\{0\}$ (b) $R-\{1\}$
(c) $R-\{-1\}$ (d) $R-\{-1, 1\}$

17. In ΔABC, orthocentre is (6,10) and circumcentre is (2, 3) and equation of side BC is $2x + y = 17$. Then the radius of the circumcircle of ΔABC is

(a) 4 (b) 5 (c) 2 (d) 3

18. An equilateral triangle has its centroid at origin and one side is $x + y = 1$. The equations of the other sides are

(a) $y+1=(2\pm\sqrt{3})(x+1)$
(b) $y+1=(2\pm\sqrt{3})x,\ y+1=(3\pm\sqrt{3})x$
(c) $y+1=(3\pm\sqrt{3})(x-1),\ y+1=\sqrt{3}x$
(d) $y\pm1=(3\pm\sqrt{3})(x-1),\ y+1=\frac{\sqrt{3}-1}{\sqrt{3}+1}(x+1)$

19. From origin, chords are drawn to the circle $x^2 + y^2 - 2y = 0$. The locus of the middle points of these chords is

(a) $x^2 + y^2 - y = 0$ (b) $x^2 + y^2 - x = 0$
(c) $x^2 + y^2 - 2x = 0$ (d) $x^2 + y^2 - x - y = 0$

20. The locus of the mid points of the chords of the circle $x^2 + y^2 - 2x - 2y - 2 = 0$ which makes an angle of 120° at the centre is

(a) $x^2 + y^2 - 2x - 2y + 1 = 0$
(b) $x^2 + y^2 + x + y + 1 = 0$
(c) $x^2 + y^2 - 2x - 2y - 1 = 0$
(d) $x^2 + y^2 + x - y - 1 = 0$

21. A straight line through $A(6, 8)$ meets the curve $2x^2 + y^2 = 2$ at B and C. P is a point on BC such that AB, AP, AC are in H.P., then the minimum distance of the origin from the locus of 'P' is

(a) $\frac{1}{\sqrt{52}}$ (b) $\frac{5}{\sqrt{52}}$ (c) $\frac{10}{\sqrt{52}}$ (d) $\frac{15}{\sqrt{52}}$

22. Consider the parabola $x^2 + 4y = 0$. Let $P(a, b)$ be any fixed point inside the parabola and let S be the focus of the parabola. Then the minimum value at $SQ + PQ$ as point Q moves on the parabola is

(a) $|1 - a|$ (b) $|ab| + 1$
(c) $\sqrt{a^2+b^2}$ (d) $1 - b$

23. If a variable tangent of the circle $x^2 + y^2 = 1$ intersect the ellipse $x^2 + 2y^2 = 4$ at P and Q, then the locus of the points of intersection of the tangents at P and Q is

(a) a circle of radius 2 units
(b) a parabola with focus as (2, 3)
(c) an ellipse with eccentricity $\frac{\sqrt{3}}{4}$
(d) an ellipse with length of latus rectum as 2 units

24. A variable straight line of slope 4 intersects the hyperbola $xy = 1$ at two points. The locus of the point which divides the line segment between these two points in the ratio 1 : 2 is

(a) $16x^2 + 10xy + y^2 = 2$ (b) $16x^2 - 10xy + y^2 = 2$
(c) $16x^2 + 10xy + y^2 = 4$ (d) $16x^2 - 10xy + y^2 = 4$

25. If the tangent drawn at $P\left(t=\frac{\pi}{4}\right)$ to the curve $x = \sec^2 t$, $y = \cot t$ meets the curve again at R, then PR =

(a) $\frac{3\sqrt{5}}{2}$ (b) $\frac{2\sqrt{5}}{3}$ (c) $\frac{5\sqrt{5}}{4}$ (d) $\frac{4\sqrt{5}}{5}$

26. The maximum area of a rectangle whose two consecutive vertices lie on the x-axis and another two lie on the curve $y = e^{-|x|}$ is equal to

(a) $2e$ sq. units (b) $2/e$ sq. units
(c) e sq. units (d) $1/e$ sq. units

27. $\int_0^{4/\pi}\left(3x^2\cdot\sin\frac{1}{x}-x\cdot\cos\frac{1}{x}\right)dx=$

(a) $\frac{8\sqrt{2}}{\pi^3}$ (b) $\frac{24\sqrt{2}}{\pi^3}$ (c) $\frac{32\sqrt{2}}{\pi^3}$ (d) $\frac{32\sqrt{2}}{\pi}$

28. The general solution of the differential equation $\frac{dy}{dx}=y\tan x-y^2\sec x$ is

(a) $\tan x = (c + \sec x)y$ (b) $\sec y = (c + \tan y)x$
(c) $\sec x = (c + \tan x)y$ (d) none of these

29. For each real number x such that $-1 < x < 1$, let $A(x)$ be the matrix $(1-x)^{-1}\begin{bmatrix}1 & -x\\ -x & 1\end{bmatrix}$ and $z=\frac{x+y}{1+xy}$. Then,

(a) $A(z) = A(x) + A(y)$ (b) $A(z) = A(x)\,[A(y)]^{-1}$
(c) $A(z) = A(x)\cdot A(y)$ (d) $A(z) = A(x) - A(y)$

30. A and B are two non singular matrices so that $A^6 = I$ and $AB^2 = BA$ $(B \ne I)$. A value of K so that $B^K = I$ is

(a) 31 (b) 32 (c) 63 (d) 64

SOLUTIONS

1. (c) : $a^2 + b^2 + c^2 = 4R^2(\sin^2A + \sin^2B + \sin^2C)$
$= 2R^2(1 - \cos2A + 1 - \cos2B + 1 - \cos2C)$
$=2R^2\left[3-\left(\cos\frac{2\pi}{7}+\cos\frac{4\pi}{7}+\cos\frac{8\pi}{7}\right)\right]$

$$= 2R^2\left[3-\left(\cos\frac{2\pi}{7}+\cos\frac{4\pi}{7}+\cos\frac{6\pi}{7}\right)\right]$$

$$= 2R^2\left[3-\left(2\sin\frac{\pi}{7}\cdot\cos\frac{2\pi}{7}+2\sin\frac{\pi}{7}\cdot\cos\frac{4\pi}{7}+2\sin\frac{\pi}{7}\cdot\cos\frac{6\pi}{7}\right)\frac{1}{2\sin\frac{\pi}{7}}\right]$$

$$= 2R^2\left[3-\frac{1}{2\sin\frac{\pi}{7}}\left(\sin\frac{3\pi}{7}-\sin\frac{\pi}{7}+\sin\frac{5\pi}{7}-\sin\frac{3\pi}{7}+\sin\frac{7\pi}{7}-\sin\frac{5\pi}{7}\right)\right]$$

$$= 2R^2\left[3+\frac{1}{2}\right] = 7R^2$$

2. (d) : $f(x) = (\sin^2 x + \cos^2 x)^3 - 3\sin^2 x\cos^2 x(\sin^2 x + \cos^2 x)$

$$= 1-\frac{3}{4}\sin^2 2x$$

$\because$ Range of $\sin^2 2x$ is $[0, 1]$

$\therefore$ Range of $f(x)$ is $\left[\frac{1}{4}, 1\right]$.

Note : Certain questions are better done by avoiding derivatives. Derivatives is one of the tools to determine extrema.

3. (d) : Use "$f(x) = x$ has non real roots $\Rightarrow f(f(x)) = x$ also has non-real roots".

4. (d) : Given, $f(x) = x - [x], x \in R - \{0\}$

Now, $f(x)+f\left(\frac{1}{x}\right)=1 \Rightarrow x-[x]+\frac{1}{x}-\left[\frac{1}{x}\right]=1$

$$\Rightarrow \left(x+\frac{1}{x}\right)-\left([x]+\left[\frac{1}{x}\right]\right)=1$$

$$\Rightarrow \left(x+\frac{1}{x}\right)=[x]+\left[\frac{1}{x}\right]+1 \quad ...(i)$$

Clearly, R.H.S. is an integer $\therefore$ L. H. S. is also an integer.

Let $x+\frac{1}{x}=k$ (an integer) $\Rightarrow x^2-kx+1=0$

$$\therefore x=\frac{k\pm\sqrt{k^2-4}}{2}$$

For real values of x, $k^2 - 4 \geq 0 \Rightarrow k \geq 2$ or $k \leq -2$

Also, $k = 2$ and -2 does not satisfy equation (i)

$\therefore$ The equation (i) will have solutions if $k > 2$ or $k < -2, k \in Z$.

Hence, equation (i) has infinite number of solutions.

5. (a) : We know that, $\frac{z_2-z_1}{z_4-z_1}=\left|\frac{z_2-z_1}{z_4-z_1}\right|e^{\frac{i\pi}{2}}=i$

$[\because |z_2 - z_1| = |z_4 - z_1|]$

$\Rightarrow (z_4 - z_1)^2 + (z_2 - z_1)^2 = 0$ ...(i)

Similarly, $\frac{z_4-z_3}{z_2-z_3}=i$

$\Rightarrow (z_4 - z_3)^2 + (z_2 - z_3)^2 = 0$ ...(ii)

On adding (i) and (ii), we get

$$2[z_1^2+z_2^2+z_3^2+z_4^2-z_1z_2-z_4z_1-z_4z_3-z_2z_3]=0$$

$$\Rightarrow z_1^2+z_2^2+z_3^2+z_4^2=z_1z_2+z_2z_3+z_3z_4+z_4z_1$$

6. (c)

7. (b) : $\frac{a+b}{2}=2\sqrt{ab} \Rightarrow a+b-4\sqrt{ab}=0$

$\Rightarrow \frac{a}{b}+1-4\sqrt{\frac{a}{b}}=0$ (Dividing by b)

$$\Rightarrow \left(\sqrt{\frac{a}{b}}\right)^2-4\sqrt{\frac{a}{b}}+1=0$$

$$\therefore \sqrt{\frac{a}{b}}=\frac{4\pm2\sqrt{3}}{2}=(2\pm\sqrt{3}) \Rightarrow \frac{a}{b}=\frac{2+\sqrt{3}}{2-\sqrt{3}}$$

8. (b) : Let $u_k=\frac{k+2}{k(k+1)(k+3)}$

$$=\frac{(k+2)^2}{k(k+1)(k+2)(k+3)}=\frac{k^2+4k+4}{k(k+1)(k+2)(k+3)}$$

$$=\frac{k(k+1)+3k+4}{k(k+1)(k+2)(k+3)}$$

$$=\frac{1}{(k+2)(k+3)}+\frac{3}{(k+1)(k+2)(k+3)}+\frac{4}{k(k+1)(k+2)(k+3)}$$

$$=\left(\frac{1}{k+2}-\frac{1}{k+3}\right)-\frac{3}{2}\left[\frac{1}{(k+2)(k+3)}-\frac{1}{(k+1)(k+2)}\right]-\frac{4}{3}\left[\frac{1}{(k+1)(k+2)(k+3)}-\frac{1}{k(k+1)(k+2)}\right]$$

Now, put $k = 1, 2, 3, ..., n$ and add. Thus

$S_n = u_1 + u_2 + ... + u_n$

$$=\left(\frac{1}{3}-\frac{1}{n+3}\right)-\frac{3}{2}\left[\frac{1}{(n+2)(n+3)}-\frac{1}{2\cdot3}\right]-\frac{4}{3}\left[\frac{1}{(n+1)(n+2)(n+3)}-\frac{1}{1\cdot2\cdot3}\right]$$

Therefore $\lim_{n\to\infty} S_n=\frac{1}{3}+\frac{3}{12}+\frac{4}{18}=\frac{29}{36}$

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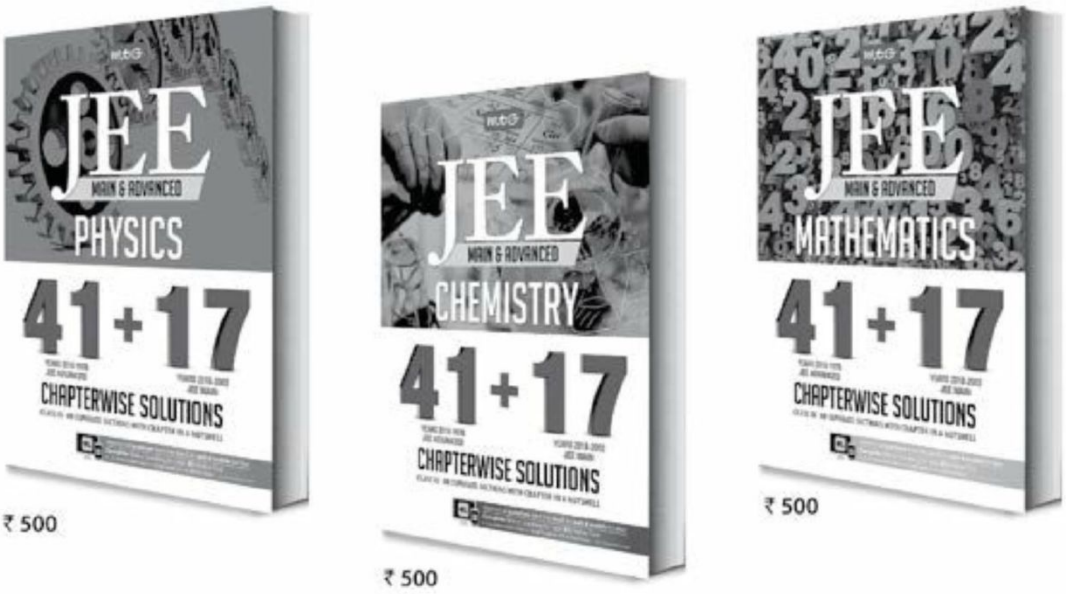

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9. (d):

$$\lim_{x\to\infty} x^{1/3}\left\{(x+1)^{1/3}+(x-1)^{1/3}\right\}\left\{(x+1)^{1/3}-(x-1)^{1/3}\right\}$$

$$=\lim_{x\to\infty}\frac{x^{1/3}\left\{(x+1)^{1/3}+(x-1)^{1/3}\right\}2}{\left\{(x+1)^{2/3}+(x^2-1)^{1/3}+(x-1)^{2/3}\right\}}$$

$$=\lim_{x\to\infty}\frac{2\cdot x^{2/3}\left\{\left(1+\frac{1}{x}\right)^{1/3}+\left(1-\frac{1}{x}\right)^{1/3}\right\}}{x^{2/3}\left\{\left(1+\frac{1}{x}\right)^{2/3}+\left(1-\frac{1}{x^2}\right)^{1/3}+\left(1-\frac{1}{x}\right)^{2/3}\right\}}$$

$$=\frac{2\times2}{3}=\frac{4}{3}$$

10. (b): We have, $\lim_{x\to\infty}\sqrt{x+\sqrt{x+\sqrt{x}}}-\sqrt{x}$

$$=\lim_{x\to\infty}\frac{\sqrt{x+\sqrt{x}}}{\sqrt{x+\sqrt{x+\sqrt{x}}}+\sqrt{x}}=\lim_{x\to\infty}\frac{\sqrt{1+\frac{1}{\sqrt{x}}}}{\sqrt{1+\sqrt{\frac{1}{x}+\frac{1}{\sqrt{x^3}}}}+1}=\frac{1}{2}$$

11. (a): $\sin^{-1}x-\cos^{-1}x=\frac{\pi}{2}-\cos^{-1}x-\cos^{-1}x$

$$=\frac{\pi}{2}-2\cos^{-1}x$$

Now, $0\le\cos^{-1}x\le\pi\Rightarrow 0\le 2\cos^{-1}x\le 2\pi$

$\Rightarrow\ -2\pi\le -2\cos^{-1}x\le 0\ \Rightarrow\ \frac{-3\pi}{2}\le\frac{\pi}{2}-2\cos^{-1}x\le\frac{\pi}{2}$

$\therefore$ Range $=\left[\frac{-3\pi}{2},\frac{\pi}{2}\right]$

12. (b): Change x to $10-x$ to obtain
$f(20-x)=f(x)$

We have, $f(20-x)=-f(20+x)\ \Rightarrow\ f(x)=-f(20+x)$

Now changing x to $20+x$, we get

$f(20+x)=-f(40+x)\ \Rightarrow\ -f(x)=-f(40+x)$

$\Rightarrow\ f(x)=f(40+x)$, so f is periodic

Again, $f(-x)=-f(20-x)=-f(x)$

Thus, f is odd

13. (a): L.C.M. $(p, q)=2^2\cdot3^4\cdot5^2$

Let $p=2^{a_1}\cdot3^{b_1}\cdot5^{c_1}$ and $q=2^{a_2}\cdot3^{b_2}\cdot5^{c_2}$

$\Rightarrow\ \max\{a_1, a_2\}=2\ \Rightarrow$ 5 ways

$\Rightarrow\ \max\{b_1, b_2\}=4\Rightarrow 9$ ways

$\Rightarrow\ \max\{c_1, c_2\}=2\Rightarrow 5$ ways

$\therefore\ K=3^2\cdot5^2$ can be expressed as $1\cdot3^25^2, 3^2\cdot5^2$

14. (b)

15. (b): $\frac{r}{R}=\cos A+\cos B+\cos C-1$

$$\Rightarrow\ \frac{1}{8}=2\cos\frac{A+B}{2}\cos\frac{A-B}{2}-1+\cos C$$

$$\Rightarrow\ \frac{1}{8}=\sin\frac{C}{2}-2\sin^2\frac{C}{2}\ \Rightarrow\ \sin\frac{C}{2}=\frac{1}{4}$$

$\therefore\ \cos C=1-\frac{1}{8}=\frac{7}{8}$. So, $\frac{1+\cos C}{1-\cos C}=\frac{1+7/8}{1-7/8}=15$

16. (d): We have, $f(x)=\begin{cases}\tan^{-1}x, & |x|\le1\\ \frac{1}{2}(|x|-1), & |x|>1\end{cases}$

$$\Rightarrow\ f(x)=\begin{cases}\frac{1}{2}(-x-1), & x<-1\\ \tan^{-1}x, & -1\le x\le1\\ \frac{1}{2}(x-1), & x>1\end{cases}$$

Clearly, L.H.L. at $(x=-1)=\lim_{h\to0}f(-1-h)=0$

R.H.L. at $(x=-1)=\lim_{h\to0}f(-1+h)$

$=\lim_{h\to0}\tan^{-1}(-1+h)=\frac{-\pi}{4}$

$\therefore$ L.H.L. $\ne$ R.H.L.

So, $f(x)$ is discontinuous at $x=-1$.

Also we can prove in the same way, that $f(x)$ is discontinuous at $x=1$

$\therefore$ $f'(x)$ can not be found for $x=\pm1$ or domain of $f'(x)=R-\{-1, 1\}$.

17. (b): Image of orthocentre of ΔABC w.r.t. BC lies on the circle.

18. (a): Third vertex A lies on $x-y=0$ and in III quadrant

Perpendicular distance from (0, 0) to $x+y=1$ is $\frac{1}{\sqrt{2}}$

$\therefore\ AO=\sqrt{2}\ \Rightarrow\ A(-1, -1)$

If m is the slope of other side,

then, $\tan60^\circ=\left|\frac{m+1}{1-m}\right|\ \Rightarrow\ m=2\pm\sqrt{3}$

19. (a): Equation of chord with middle point (x_1, y_1) is given by $T=S_1$

i.e., $xx_1+yy_1-(y+y_1)=x_1^2+y_1^2-2y_1$ ...(i)

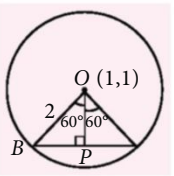
$\because$ (i) passes through (0, 0)

$\therefore$ Required locus is $x^2+y^2-y=0$

20. (a): Centre $\equiv$ (1, 1)
and radius = 2 = OB

In ΔOBP, $\angle OBP=30^\circ$

$\sin30^\circ=\frac{OP}{2}\ \Rightarrow\ OP=1$



Since, $OP=1\ \Rightarrow\ x^2+y^2-2x-2y+1=0$

21. (a) : $(6 + r\cos\theta, 8 + r\sin\theta)$ lies on $2x^2 + y^2 = 2$

$\Rightarrow (2\cos^2\theta + \sin^2\theta)r^2 + 2(12\cos\theta + 8\sin\theta)r + 134 = 0$

AB, AP, AC are in H.P.

$\Rightarrow \frac{2}{r} = \frac{AB + AC}{AB \cdot AC} \Rightarrow \frac{1}{r} = -\frac{(6\cos\theta + 4\sin\theta)}{67}$

$\Rightarrow 6x + 4y - 1 = 0$

$\therefore$ Minimum distance from origin $= \frac{1}{\sqrt{52}}$

22. (d) : Let foot of perpendicular from Q to the directrix be N.

Now, $SQ + PQ = QN + PQ$ is minimum if

P, Q and N are collinear

So, minimum value of $SQ + PQ = PN = 1 - b$

23. (d) : We have, $x^2 + y^2 = 1$; $x^2 + 2y^2 = 4$

Let $R(x_1, y_1)$ is point of intersection of tangents drawn at P, Q to ellipse

$\Rightarrow PQ$ is chord of contact of $R(x_1, y_1)$

$\Rightarrow xx_1 + 2yy_1 - 4 = 0$ ...(i)

(i) touches circle if $r^2(l^2 + m^2) = n^2$

$\Rightarrow 1(x_1^2 + 4y_1^2) = 16000$

$\therefore$ Required locus is ellipse $x^2 + 4y^2 = 16$ with $e = \frac{\sqrt{3}}{2}$ and latus rectum = 2 units

24. (a) : Let $P(h, k)$ be the required point.

$\therefore y - k = 4(x - h)$ ...(i)

Let it meets $xy = 1$ ...(ii) at $A(x_1, y_1)$ and $B(x_2, y_2)$

$\Rightarrow x_1 + x_2 = \frac{4h - k}{4}, x_1x_2 = -\frac{1}{4}$

Also, $\frac{2x_1 + x_2}{3} = h \Rightarrow x_1 = \frac{8h + k}{4}, x_2 = -\frac{2h + k}{2}$

$\therefore$ Required locus is $16x^2 + 10xy + y^2 = 2$.

25. (a) : We have, $x = \sec^2 t, y = \cot t$ ...(i)

At $t = \frac{\pi}{4}$, $x = 2$, $y = 1 \Rightarrow P$ is $(2, 1)$

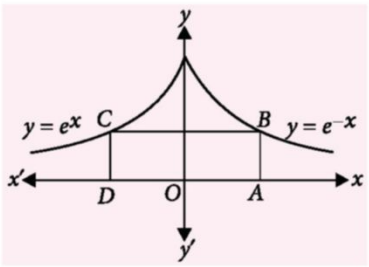
$$\left[\frac{dy}{dx}\right]_{t=\frac{\pi}{4}} = \left[\frac{-\text{cosec}^2 t}{2\sec t \cdot \sec t \cdot \tan t}\right]_{t=\frac{\pi}{4}} = -\frac{1}{2}$$

$\therefore$ Tangent at $P(2, 1)$ is, $y = \frac{4 - x}{2}$ ...(ii)

Eliminating 't' using (i) and (ii), we get

$R \equiv \left(5, -\frac{1}{2}\right) \therefore PR = \frac{3}{2}\sqrt{5}$

26 (b) :



Let the rectangle be $ABCD$.

$A \equiv (t, 0), B \equiv (t, e^{-t}), C \equiv (-t, e^{-t}), D = (-t, 0)$

Area of $ABCD = 2te^{-t} = f(t)$ (say)

Now, $\frac{df}{dt} = 2(t(-e^{-t}) + e^{-t}) = 2e^{-t}(1 - t)$

$$\frac{df}{dt} > 0 \Rightarrow t \in (0,1), \frac{df}{dt} < 0 \Rightarrow t \in (1, \infty)$$

$\Rightarrow t = 1$ is point of maxima.

$\therefore$ Maximum area $= f(1) = \frac{2}{e}$.

27. (c)

28. (c) : We have, $\frac{dy}{dx} = y\tan x - y^2 \sec x$

$\Rightarrow \frac{1}{y^2}\frac{dy}{dx} - \frac{1}{y}\tan x = -\sec x$

Putting $\frac{1}{y} = v \Rightarrow \frac{-1}{y^2}\frac{dy}{dx} = \frac{dv}{dx}$, we obtain

$\frac{dv}{dx} + \tan x \cdot v = \sec x$, which is linear D.E.

I.F. $= e^{\int \tan x dx} = e^{\log \sec x} = \sec x$

$\therefore$ The solution is given by

$$v\sec x = \int \sec^2 x dx + c \Rightarrow \frac{1}{y}\sec x = \tan x + c$$

$\Rightarrow \sec x = y(c + \tan x)$

29. (c) : $A(z) = A\left(\frac{x + y}{1 + xy}\right)$

$$= \left[\frac{1 + xy}{(1 - x)(1 - y)}\right]\begin{bmatrix} 1 & -\left(\frac{x + y}{1 + xy}\right) \\ -\left(\frac{x + y}{1 + xy}\right) & 1 \end{bmatrix}$$

$\therefore A(x) \cdot A(y) = A(z)$

30. (c) : We have, $AB^2 = BA \Rightarrow A^5(AB^2) = A^5BA$

$\Rightarrow B^2 = A^5BA$

$\Rightarrow B^4 = (A^5BA)(A^5BA) = A^5B^2A = A^5(A^5BA)A$

$\Rightarrow B^4 = A^4BA^2$

$\Rightarrow B^8 = (A^4BA^2)(A^4BA^2) = A^4B^2A^2 = A^4(A^5BA)A^2$

$\Rightarrow B^8 = A^3\,BA^3$

$\Rightarrow B^{16} = (A^3BA^3)(A^3BA^3) = A^3B^2A^3 = A^3(A^5BA)A^3 = A^2BA^4$

$\Rightarrow B^{32} = (A^2BA^4)(A^2BA^4) = A^2B^2A^4 = A^2(A^5BA)A^4 = ABA^5$

$\Rightarrow B^{64} = (ABA^5)(ABA^5) = AB^2A^5 = A(A^5BA)A^5 = B$

$\Rightarrow B^{63} = I$

CONCEPT MAP

PERMUTATIONS AND COMBINATIONS

Class XI

Properties

- ${}^nP_n = n! = n(n-1)\cdot(n-2)\ldots 3\cdot 2\cdot 1 = {}^nP_{n-1}$
- ${}^nP_1 = n$
- ${}^nP_0 = \dfrac{n!}{(n-0)!} = 1$
- ${}^nP_r = n\cdot{}^{n-1}P_{r-1} = n(n-1)\,{}^{n-2}P_{r-2}$
  $= n(n-1)(n-2)\,{}^{n-3}P_{r-3}$ and so on
- ${}^{n-1}P_r + r\cdot{}^{n-1}P_{r-1} = {}^nP_r$
- $\dfrac{{}^nP_r}{{}^nP_{r-1}} = n-r+1$

Circular Permutations

(i) Arrangement of n different things taken all at a time in form of circle is

- $(n-1)!$, if sense matter.
- $1/2(n-1)!$, if sense doesn't matter

(ii) Number of circular permutations of n dissimilar things taken r at a time

$= \dfrac{{}^nP_r}{r}$ if clockwise and anticlockwise orders are considered as different.

$= \dfrac{{}^nP_r}{2r}$ if clockwise and anticlockwise order is considered as same.

Factorial Notation

Product of first n natural numbers is denoted by $n!$

i.e., $n! = n(n-1)(n-2)\ldots 3\cdot 2\cdot 1$

Fundamental Principle of Counting

In an operation A can be performed in m different ways and another operation B can be performed in n different ways, then

- Both the operations can be performed in $m \times n$ ways.
- Either of the two operations can be performed in $(m+n)$ ways.

Permutations

Arranging r objects out of n different things

- When repetition is not allowed $= {}^nP_r = \dfrac{n!}{(n-r)!}$, where $0 \le r \le n$
- When repetition is allowed $= n^r$

Restricted Permutations

The number of ways in which r objects can be arranged from n dissimilar objects if k particular objects are

- Always included (or never excluded)
  $= {}^{n-k}C_{r-k}\, r! = {}^rP_k\,{}^{n-k}P_{r-k}$.
- Always excluded (never included)
  $= {}^{n-k}C_r\, r! = {}^{n-k}P_r$

Distributions into Groups

Distribution of n distinct things into r groups $G_1, G_2, \ldots, G_r$ containing $P_1, P_2, \ldots, P_r$ elements respectively.

- Groups are distinct : $\dfrac{n!}{P_1!\,P_2!\ldots P_r!}\,r!$
- Groups are identical : $\dfrac{n!}{P_1!\,P_2!\ldots P_r!}$

De-arrangements

Any change in the existing order of things is called De-arrangement. If m things are arranged in a row, the number of ways in which they can be dearranged so that none of them occupies its original place (no one of them occupies the place assigned to it)

$$= m!\left[1-\frac{1}{1!}-\frac{1}{1!}+\frac{1}{2!}-\frac{1}{3!}+\ldots+(-1)^m\frac{1}{m!}\right]$$

$$= m!\sum_{r=0}^{m}(-1)^r\frac{1}{r!}, \text{ we denote it by } D(m)$$

$$= {}^mP_m - {}^mP_{m-1} + {}^mP_{m-2} - \ldots + (-1)^m\,{}^mP_0$$

Combinations

- Selecting r objects out of n different things given by
  ${}^nC_r = \dfrac{n!}{r!\,(n-r)!}, 0 \le r \le n$

Properties

- ${}^nP_r = {}^nC_r\, r!,\ 0 \le r \le n$
- For $0 \le r \le n$, ${}^nC_r = {}^nC_{n-r}$
- For $1 \le r \le n$, ${}^nC_r + {}^nC_{r-1} = {}^{n+1}C_r$
- ${}^nC_a = {}^nC_b \Rightarrow a = b$ or $n = a+b$
- ${}^nC_0 + {}^nC_1 + \ldots + {}^nC_n = 2^n$

Restricted Combinations

The number of ways in which r objects can be selected from n dissimilar objects if k particular objects are

- Always included $= {}^{n-k}C_{r-k} = {}^{n-k}C_{n-r}$
- never included (Always excluded) $= {}^{n-k}C_r$

DEFINITE INTEGRALS AND APPLICATION OF INTEGRALS

CONCEPT MAP

Class XII

DEFINITE INTEGRALS

Fundamental Theorem of Calculus

- **First Fundamental Theorem :** Let $f(x)$ be a continuous function on the closed interval $[a, b]$ and let $A(x)$ be the area function. Then $A'(x) = f(x)$, for all $x \in [a, b]$.
- **Second Fundamental Theorem :** Let $f(x)$ be a continuous function on the closed interval $[a, b]$ and $F(x)$ be an integral of $f(x)$, then $\int_a^b f(x)dx = [F(x)]_a^b = F(b) - F(a)$

For any two values a and b, we have $\int_a^b f(x)\,dx = [F(x) + c]_a^b$
$= F(b) - F(a)$
Here, $F(x)$ is anti derivative of function $f(x)$.

Solving by Substitution

When definite integral is to be found by substitution, change the lower and upper limits of integration. If substitution is $t = f(x)$ and lower limit of integration is a and upper limit is b, then new lower and upper limits will be $f(a)$ and $f(b)$ respectively.

Limit of Sum

$\int_a^b f(x)\,dx = \lim_{h\to 0} h[f(a) + f(a+h) + \ldots + f(a+(n-1)h)]$,

where $h = \frac{b-a}{n} \to 0$ as $n \to \infty$

Properties

- $\int_a^b f(x)dx = \int_a^b f(t)\,dt$
- $\int_a^b f(x)dx = -\int_b^a f(x)dx$

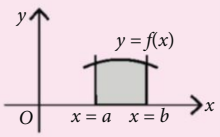
  In particular $\int_a^a f(x)dx = 0$
- $\int_a^b f(x)dx = \int_a^c f(x)dx + \int_c^b f(x)dx$,

  where $a < c < b$
- $\int_a^b f(x)dx = \int_a^b f(a+b-x)dx$
- $\int_0^a f(x)dx = \int_0^a f(a-x)dx$
- $\int_{-a}^a f(x)dx = \begin{cases} 0 & \text{, if } f(-x) = -f(x) \\ 2\int_0^a f(x)dx & \text{, if } f(-x) = f(x) \end{cases}$
- $\int_0^{2a} f(x)dx = \int_0^a f(x)dx + \int_0^a f(2a-x)dx$
- $\int_0^{2a} f(x)dx = \begin{cases} 2\int_0^a f(x)dx, & \text{if } f(2a-x) = f(x) \\ 0, & \text{if } f(2a-x) = -f(x) \end{cases}$

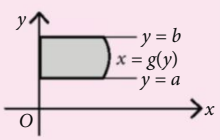
APPLICATION OF INTEGRALS

Area Under Simple Curves

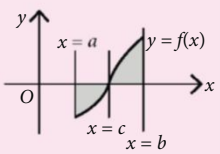
- Area $= \int_a^b y\,dx$
  $= \int_a^b f(x)dx$ (where $b > a$)



- Area $= \int_a^b x\,dy$
  $= \int_a^b g(y)dy$ (where $b > a$)



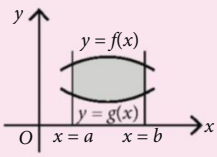
- Area $= \left|\int_a^c f(x)dx\right| + \int_c^b f(x)dx$



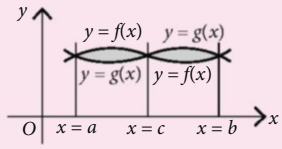
Area Between Two Curves

- Area $= \int_a^b [f(x) - g(x)]dx$,

  $f(x) \geq g(x)$ in $[a, b]$

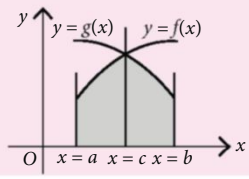


- Area $= \int_a^c [f(x) - g(x)]dx$
  $+ \int_c^b [g(x) - f(x)]dx$



where $f(x) \geq g(x)$ in $[a, c]$ and $f(x) \leq g(x)$ in $[c, b]$

- Area $= \int_a^c f(x)dx + \int_c^b g(x)dx$



JEE Advanced Exam on 19th May, 2019

JEE WORK OUTS

One or More Than One Option(s) Correct Type

1. The equation $16x^2 - 3y^2 - 32x - 12y - 44 = 0$ represents a hyperbola with

(a) length of the transverse axis $= 2\sqrt{3}$

(b) length of the conjugate axis = 8

(c) centre at (1, –2) (d) eccentricity $= \sqrt{19}$

2. Which of the following functions is(are) periodic?

(a) Sgn (e^{-x}) (b) $\sin x + |\sin x|$

(c) min $(\sin x, |x|)$ (d) none of these

3. The number of solutions for the equation

$$2\sin^{-1}\sqrt{x^2-x+1} + \cos^{-1}\sqrt{x^2-x} = \frac{3\pi}{2} \text{ is}$$

(a) 1 (b) 2 (c) 3 (d) infinite

4. The equation $2\sin\left(\frac{x}{2}\right)\cos^2 x - 2\sin\left(\frac{x}{2}\right)\sin^2 x = \cos^2 x - \sin^2 x$ has a root for which

(a) $\sin 2x = 1$ (b) $\sin 2x = -1$

(c) $\cos x = \frac{1}{2}$ (d) $\cos 2x = -\frac{1}{2}$

5. A triangle is inscribed in a circle. The vertices of the triangle divide the circle into the arcs of length 3, 4 and 5 units, then the area of the triangle is equal to

(a) $\frac{9\sqrt{3}(1+\sqrt{3})}{\pi^2}$ sq.units (b) $\frac{9\sqrt{3}(\sqrt{3}-1)}{\pi^2}$ sq.units

(c) $\frac{9\sqrt{3}(1+\sqrt{3})}{2\pi^2}$ sq.units (d) $\frac{9\sqrt{3}(\sqrt{3}-1)}{2\pi^2}$ sq.units

6. Let $[\cdot]$ denote the greatest integer function and $f(x) = [\tan^2 x]$. Then which of the following is not true?

(a) $\lim_{x\to 0} f(x)$ does not exist

(b) $f(x)$ is continuous at $x = 0$

(c) $f(x)$ is not differentiable at $x = 0$

(d) none of these.

7. If $\int \frac{dx}{\cos(x-a)\cos(x-b)} = \frac{1}{A}[\log_e(f(x))/\log_e g(x)] + c$, then

(a) $A = \sin(a-b)$ (b) $f(x) = \sec(x-b)$

(c) $g(x) = \sec(x-a)$ (d) none of these.

8. If $\cos\theta + \cos\phi = \alpha$, $\cos 2\theta + \cos 2\phi = \beta$ and $\cos 3\theta + \cos 3\phi = \gamma$, then

(a) $\cos^2\theta + \cos^2\phi = 1 + \frac{\beta}{2}$

(b) $\cos\theta \cdot \cos\phi = \frac{\alpha^2}{2} - \frac{\beta+2}{4}$

(c) $2\alpha^3 + \gamma = 3\alpha(1+\beta)$

(d) $\alpha + \beta + \gamma = 3\alpha\beta\gamma$

9. If $f(x) = x(\sqrt{x} - \sqrt{x+1})$, then

(a) f is continuous but not differentiable at $x = 0$

(b) f is differentiable at $x = 0$

(c) f is not differentiable at $x = 0$

(d) f is continuous at $x = 0$

10. If A and B are two events such that $P(A) = 1/2$ and $P(B) = 2/3$, then

(a) $P(A \cup B) \geq 2/3$ (b) $P(A \cap B') \leq 1/3$

(c) $1/6 \leq P(A \cap B) \leq 1/2$ (d) $1/6 \leq P(A' \cap B) \leq 1/2$

11. If a circle passes through the point $\left(3, \sqrt{\frac{7}{2}}\right)$ and touches $x + y = 1$ and $x - y = 1$, then the centre of the circle is

(a) (4, 0) (b) (4, 2) (c) (6, 0) (d) (7, 9)

12. If N is the number of positive integral solutions of $x_1 x_2 x_3 x_4 = 770$, then

(a) N is divisible by 4 distinct primes

(b) N is a perfect square

(c) N is a perfect fourth power

(d) N is a perfect 8th power

13. In a triangle ABC, if $a=\sqrt{6}, c=\sqrt{8}$ and $A=\frac{\pi}{4}$, its area is

(a) $2-\sqrt{2}$ (b) $3-\sqrt{2}$ (c) $2+\sqrt{2}$ (d) $3+\sqrt{2}$

14. The values of 'x' satisfying the equation $\sin^{-1}|\sin x| = \sqrt{\sin^{-1}|\sin x|}$ are

(a) $2n\pi - 1, n \in Z$ (b) $2n\pi + 1, n \in Z$

(c) $(2n+1)\pi - 1, n \in Z$ (d) $(2n+1)\pi + 1, n \in Z$

15. Let $w=\frac{\sqrt{3}+i}{2}$ and $P=\{w^n : n = 1, 2, 3, ...\}$. Further $H_1=\left\{z \in C : \text{Re}\, z > \frac{1}{2}\right\}$ and $H_2 = \left\{z \in C : \text{Re}\, z < -\frac{1}{2}\right\}$ where C is the set of all complex numbers. If $z_1 \in P \cap H_1$, $z_2 \in P \cap H_2$ and O represents the origin, then $\angle z_1 O z_2 =$

(a) $\frac{\pi}{2}$ (b) $\frac{\pi}{6}$ (c) $\frac{2\pi}{3}$ (d) $\frac{5\pi}{6}$

Comprehension Type

Paragraph for Q. No. 16 to 18

Given a sequence $t_1, t_2, ...$ if it is possible to find a function $f(r)$ such that $t_r = f(r+1) - f(r)$, then

$$\sum_{r=1}^{n} t_r = f(n+1) - f(1)$$

16. If $u_1 = 1$, $u_{n+1} = 2u_n + 1$, then u_{n+1} is equal to

(a) $2^n + 1$ (b) $2^{n+1} - 1$ (c) $2^n - 2$ (d) $2^{n+1} - 2$

17. Sum of the $\sum_{r=1}^{\infty} \frac{1}{r(r+1)(r+2)}$ is

(a) 1 (b) 1/2 (c) 1/4 (d) 1/8

18. Sum of the series $\sum_{r=1}^{n} r(r+1)(r+2)$ is

(a) $4\ (^{n+3}C_2)$ (b) $6\ (^{n+3}C_4)$

(c) $n^2 - n$ (d) $n^2 + n$

Paragraph for Q. No. 19 to 21

For $n \in N$, we put $(1+x+x^2)^n = \sum_{r=0}^{2n} a_r x^r$

19. Value of $2(a_0 + a_1 + ... + a_{n-1}) + a_n$ is

(a) 2^{2n-1} (b) 3^n (c) $3^{n/2}$ (d) $(3^n - 1)/2$

20. If $a_0^2 - a_1^2 + a_2^2 - a_3^2 + ... + a_{2n}^2 = ka_n$, then k equals

(a) 1 (b) 2 (c) 1/2 (d) 0

21. If n is not a multiple of 3, and $\sum_{r=0}^{n} (-1)^r a_r (^nC_r) = k(^nC_{[n/3]})$, where $[x]$ denotes the greatest integer $\le x$, then k is equal to

(a) 1 (b) 0 (c) 3 (d) –1

Paragraph for Q. No. 22 to 24

If a source of light is placed at the fixed point of a parabola and if the parabola is reflecting surface, then the ray will bounce back in a line parallel to the axis of parabola.

22. A ray of light moving parallel to the x-axis gets reflected from a parabolic mirror whose equation is $y^2 + 10y - 4x + 17 = 0$. After reflection, the ray must pass through the point

(a) (–2, –5) (b) (–1, –5)

(c) (–3, –5) (d) (–4, –5)

23. A ray of light is coming along the line $x = 2$ from the positive direction of y-axis and strikes a concave mirror whose intersection with the x-y plane is a parabola $x^2 = 4y$, then the equation of the reflected ray after second reflection is

(a) $2x + y = 1$ (b) $2x - 2y + 2 = 0$

(c) $y - 1 = 0$ (d) none of these

24. Two rays of light coming along the line $y = 1$ and $y = -2$ from the positive direction of x-axis and strikes a concave mirror whose intersection with the x-y plane is a parabola $y^2 = x$ at A and B respectively. The reflected rays pass through a fixed point C, then the area of the triangle ABC is

(a) $\frac{21}{8}$ sq.units (b) $\frac{19}{2}$ sq.units

(c) $\frac{17}{2}$ sq.units (d) $\frac{15}{2}$ sq.units

Matrix Match Type

25. For $0 < \theta < \pi/4$, let $x=\sum_{n=0}^{\infty} (\sin\theta)^{2n}, y=\sum_{n=0}^{\infty} (\cos\theta)^{2n}$, then match the following columns :

Column-I		Column-II	
(A)	$\sum_{n=0}^{\infty} \sin^{2n}\theta \cos^{2n}\theta$	(p)	$\frac{xy^2}{xy^2-1}$
(B)	$\sum_{n=0}^{\infty} \tan^{2n}\theta$	(q)	$\frac{y}{y-x}$
(C)	$\sum_{n=0}^{\infty} \sin^{2n}\theta \cos^{4n}\theta$	(r)	$\frac{xy}{xy-1}$
(D)	$\sum_{n=0}^{\infty} \cos^{2n}\theta \sin^{4n}\theta$	(s)	$\frac{x^2y}{x^2y-1}$

(a) (A) → (q) ; (B) → (p) ; (C) → (r) ; (D) → (s)

(b) (A) → (s) ; (B) → (p) ; (C) → (q) ; (D) → (r)

(c) (A) → (r) ; (B) → (q) ; (C) → (p) ; (D) → (s)

(d) (A) → (s) ; (B) → (q) ; (C) → (p) ; (D) → (r)

26. Match the following columns :

Column-I		Column-II	
(A)	If the greatest and least values of the function $f(x)=\begin{cases}2x^2+\frac{2}{x^2}, & x\in[-2,2]-\{0\}\\ 1, & x=0\end{cases}$ are G and L respectively, then	(p)	$[G+L]=1$ where $[.]$ = greatest integer function
(B)	If the greatest and least values of the function $f(x)=x^3-6x^2+9x+1$ on $[0,2]$ are G and L respectively, then	(q)	$[G+L]=6$, where $[.]$ = greatest integer function
(C)	If the greatest and least values of the functions $f(x)=\tan^{-1}x-\frac{1}{2}\ln x$ on $\left[\frac{1}{\sqrt{3}},\sqrt{3}\right]$ are G and L respectively, then	(r)	$[G+L]=9$, where $[.]$ = greatest integer function
		(s)	$(G+L)=2$, where $(.)$ = Least integer function

(a) (A) → (s) ; (B) → (p,s) ; (C) → (q)
(b) (A) → (r) ; (B) → (q) ; (C) → (p,s)
(c) (A) → (p,q) ; (B) → (r) ; (C) → (s)
(d) (A) → (s) ; (B) → (p) ; (C) → (q)

27. Match the following columns :

Column-I		Column-II	
(A)	If the distance of any point (x, y) from origin is defined as $d(x, y) = 2\|x\| + 3\|y\|$. If perimeter and area of figure bounded by $d(x, y) = 6$ are λ units and μ sq. units respectively, then	(p)	(λ, μ) lies on $x^2 - y^2 = 64$
(B)	If the vertices of a triangle are (6, 0), (0, 6) and (6, 6). If the distance between circumcentre and orthocentre and distance between circumcentre and centroid are λ unit and μ unit respectively, then	(q)	(λ, μ) lies on $x^2 + y^2 - 6x - 6y = 0$
(C)	The ends of the hypotenuse of a right angled triangle are (6, 0) and (0, 6). If the third vertex is (λ, μ), then	(r)	(λ, μ) lies on $x^2 - 16y = 16$
		(s)	(λ, μ) lies on $x^2 - y^2 = 16$

(a) (A) → (p) ; (B) → (s) ; (C) → (q)
(b) (A) → (p,r) ; (B) → (r) ; (C) → (p)
(c) (A) → (p) ; (B) → (q) ; (C) → (r,s)
(d) (A) → (q) ; (B) → (p,q) ; (C) → (r)

Numerical Answer Type

28. If the straight lines joining the origin to the points of intersection of $2x + 3y = k$ and $3x^2 - xy + 3y^2 + 2x - 3y - 4 = 0$ are at right angle, then $6k - \frac{52}{k}$ is

29. $f(x) = 1 - x^3 - x^4 - 2x^5 = g(x) + h(x)$, where g is an even function and h is an odd function. If $h(-5) = 125\,k$ then k equals

30. Each of two boxes contains both black and white marbles, and the total number of marbles in the two boxes is 25. One marble is taken out of each box. The probability that both marbles are black is $\frac{27}{50}$. If the probability that both marbles are white is $\frac{m}{n}$, where m, n, are relatively prime positive integers, then m is

31. The value of $\lim\limits_{x\to\pi/4}\frac{8-\sqrt{2}(\cos x+\sin x)^5}{1-\sin 2x}$ is

32. The solution of the differential equation $\frac{dy}{dx}+\frac{y}{x}=x^2y^6$ is $\frac{1}{x^5y^5}=\frac{p}{qx^2}+c$, then the value of $p + q$ is

33. Let $F(x)=\int\limits_{x}^{x+3}2t(5-t)dt$. If M is the maximum value of $F(x)$, then the sum of the digits of M is

34. If A is the area bounded by $y = x^{2/3} - x^{4/3}$ and x-axis, then the value of $35A/4$ is

35. A pack contains n cards numbered from 1 to n. Two consecutive numbered cards are removed from the pack and the sum of the numbers on the remaining cards is 1224. If the smaller of the numbers on the removed cards is k, then $k - 20 =$

SOLUTIONS

1. (a,b,c) : $16x^2 - 3y^2 - 32x - 12y - 44 = 0$
$\Rightarrow 16(x^2 - 2x) - 3(y^2 + 4y) - 44 = 0$
$\Rightarrow 16\{(x - 1)^2 - 1\} - 3\{(y + 2)^2 - 4\} - 44 = 0$
$\Rightarrow 16(x - 1)^2 - 3(y + 2)^2 = 48$
$\Rightarrow \frac{(x-1)^2}{(\sqrt{3})^2}-\frac{(y+2)^2}{4^2}=1$
Now, length of transverse axis $=2\times\sqrt{3}=2\sqrt{3}$
Length of conjugate axis = 2 × 4 = 8

Centre is given by $x - 1 = 0$ and $y + 2 = 0$, *i.e.*, $(1, -2)$

Also, $4^2 = (\sqrt{3})^2(e^2 - 1) \Rightarrow e^2 - 1 = \frac{16}{3}$

$\therefore \quad e^2 = \frac{19}{3} \Rightarrow e = \sqrt{\frac{19}{3}}$

2. (a,b,c) : (a) $\text{Sgn}(x) = \begin{cases} 1, & \text{if } x > 0 \\ 0, & \text{if } x = 0 \\ -1, & \text{if } x < 0 \end{cases}$

$\therefore \quad \text{Sgn}(e^{-x}) = 1 \qquad (\because e^{-x} > 0)$

$\Rightarrow$ $\text{Sgn}(e^{-x})$ is a constant function. Hence, it is periodic.

(b) $\because$ Period of $\sin x$ is 2π and period of $|\sin x|$ is π

$\therefore$ Period of $\sin x + |\sin x|$ is L.C.M. $\{2\pi, \pi\} = 2\pi$

(c) Let $f(x) = \min \{\sin x, |x|\} = \sin x \quad [\because \sin x < |x|]$ and $\sin x$ is periodic with period 2π.

3. (b) : $\because x^2 - x + 1 = \left(x - \frac{1}{2}\right)^2 + \frac{3}{4} > 0$

For $\sin^{-1}\sqrt{x^2 - x + 1}$, $0 < x^2 - x + 1 \le 1$

or $x^2 - x \le 0$ ...(i)

For $\cos^{-1}\sqrt{x^2 - x}$, $x^2 - x \ge 0$ ...(ii)

From (i) and (ii), $x^2 - x = 0 \Rightarrow x = 0, 1$

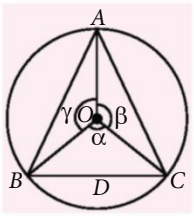
4. (a,b,c,d) : $2\sin\frac{x}{2}(\cos^2 x - \sin^2 x) - (\cos^2 x - \sin^2 x) = 0$

$\Rightarrow \quad \left(2\sin\frac{x}{2} - 1\right)(\cos^2 x - \sin^2 x) = 0$

$\Rightarrow \quad \left(2\sin\frac{x}{2} - 1\right)\cos 2x = 0 \Rightarrow \sin\frac{x}{2} = \frac{1}{2}$ and $\cos 2x = 0$

$\Rightarrow \quad x = 60°$ and $x = \pm 45°$

5. (a) : Given, arc $(BC) = 3$, arc $(CA) = 4$, arc $(AB) = 5$



Let r be the radius of the circle.

Then, $3 = r\alpha$, $4 = r\beta$, $5 = r\gamma$

Now, $3 + 4 + 5 = r(\alpha + \beta + \gamma) = r \cdot 2\pi$

$\Rightarrow \quad r = \frac{6}{\pi}$

Now, $\text{Ar}(\Delta ABC) = \text{Ar}(\Delta OBC) + \text{Ar}(\Delta OCA) + \text{Ar}(\Delta OAB)$

$= \frac{1}{2}r^2\sin\alpha + \frac{1}{2}r^2\sin\beta + \frac{1}{2}r^2\sin\gamma$

$= \frac{1}{2}r^2\left\{\sin\left(\frac{3}{r}\right) + \sin\left(\frac{4}{r}\right) + \sin\left(\frac{5}{r}\right)\right\}$

$= \frac{1}{2} \times \frac{36}{\pi^2}\left[\sin\frac{\pi}{2} + \sin\frac{2\pi}{3} + \sin\frac{5\pi}{6}\right]$

$= \frac{18}{\pi^2}\left\{1 + \frac{\sqrt{3}}{2} + \frac{1}{2}\right\} = \frac{9\sqrt{3}(1 + \sqrt{3})}{\pi^2}$ sq.units

6. (a,c) : $f(x) = [\tan^2 x] = 0$ for $-\pi/4 < x < \pi/4$. Thus $\lim_{x \to 0} f(x)$ exists and the value is 0. Moreover, it is continuous at $x = 0$. Being a constant function f is differentiable at $x = 0$ and $f'(0) = 0$.

7. (a,b,c) : Let $I = \int \frac{dx}{\cos(x - a)\cos(x - b)}$

$\Rightarrow \quad I = \frac{1}{\sin(a - b)}\int \frac{\sin(x - b - (x - a))dx}{\cos(x - a)\cos(x - b)}$

$= \frac{1}{\sin(a - b)}\int (\tan(x - b) - \tan(x - a))dx$

8. (a, b, c): $(\cos\theta + \cos\phi)^2 = \alpha^2$ ...(i)

$\Rightarrow \quad \cos^2\theta + \cos^2\phi + 2\cos\theta\cos\phi = \alpha^2$

Now, $\cos 2\theta + \cos 2\phi = \beta$

$\Rightarrow \quad (2\cos^2\theta - 1) + (2\cos^2\phi - 1) = \beta$

$\Rightarrow \quad 2(\cos^2\theta + \cos^2\phi) = \beta + 2$

$\Rightarrow \quad \cos^2\theta + \cos^2\phi = \frac{\beta}{2} + 1$ ...(ii)

From (i) and (ii), we get $\cos\theta \cdot \cos\phi = \frac{\alpha^2}{2} - \frac{\beta + 2}{4}$

Also, $\cos 3\theta + \cos 3\phi = \gamma$

$\Rightarrow \quad (4\cos^3\theta - 3\cos\theta) + (4\cos^3\phi - 3\cos\phi) = \gamma$

$\Rightarrow \quad 4(\cos^3\theta + \cos^3\phi) - 3(\cos\theta + \cos\phi) = \gamma$

$\Rightarrow \quad 4[(\cos\theta + \cos\phi)(\cos^2\theta + \cos^2\phi - \cos\theta\cos\phi)]$

$\qquad -3(\cos\theta + \cos\phi) = \gamma$

$\Rightarrow \quad 4\left[\alpha\left(\frac{\beta + 2}{2} - \frac{1}{2}\left(\alpha^2 - \frac{(\beta + 2)}{2}\right)\right)\right] - 3\alpha = \gamma$

$\therefore \quad 2\alpha^3 + \gamma = 3\alpha(1 + \beta)$

9. (b, d) : $f'(x) = \sqrt{x} - \sqrt{x + 1} + x\left(\frac{1}{2\sqrt{x}} - \frac{1}{2\sqrt{x + 1}}\right), x > 0$

$= (\sqrt{x} - \sqrt{x + 1}) + \frac{x}{2}\left(\frac{\sqrt{x + 1} - \sqrt{x}}{\sqrt{x}\sqrt{x + 1}}\right)$

Also $f'(0^+) = \lim_{h \to 0^+} \frac{h(\sqrt{h} - \sqrt{h + 1})}{h} = -1.$

Hence f is differentiable at $x = 0$ (see the definition of differentiability at the end point of an interval). This also shows f is continuous at $x = 0$.

10. (a,b,c,d) : Given, $P(A) = 1/2$, $P(B) = 2/3$

$P(A \cup B) = P(A) + P(B) - P(A \cap B) = \frac{1}{2} + \frac{2}{3} - P(A \cap B)$

$\therefore \quad P(A \cup B) = \frac{7}{6} - P(A \cap B) \ge \frac{2}{3} \Rightarrow P(A \cap B) \le \frac{7}{6} - \frac{2}{3}$

$\Rightarrow \quad P(A \cap B) \le \frac{1}{2}$ which is true. ...(i)

Now, $P(A \cap B') = P(A) - P(A \cap B) \le \frac{1}{3}$

$\Rightarrow \frac{1}{2} - P(A \cap B) \le \frac{1}{3} \Rightarrow P(A \cap B) \ge \frac{1}{2} - \frac{1}{3}$

$\therefore P(A \cap B) \ge \frac{1}{6}$ which is true. ...(ii)

From (i) and (ii), $\frac{1}{6} \le P(A \cap B) \le \frac{1}{2}$ ...(iii)

From (iii), $-\frac{1}{6} \ge -P(A \cap B) \ge -\frac{1}{2}$

or $P(B) - \frac{1}{6} \ge P(B) - P(A \cap B) \ge P(B) - \frac{1}{2}$

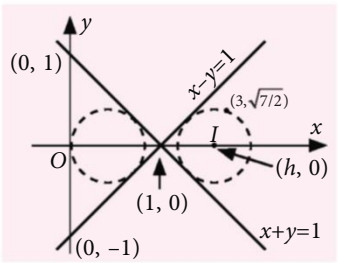
$\Rightarrow \frac{2}{3} - \frac{1}{6} \ge P(A' \cap B) \ge \frac{2}{3} - \frac{1}{2}$

$\Rightarrow \frac{1}{6} \le P(A' \cap B) \le \frac{1}{2}$

11. (a,c) : If centre is $(h, 0)$, then $\frac{|h-0-1|}{\sqrt{2}} = \sqrt{(h-3)^2 + \frac{7}{2}}$

$\therefore \quad h = 6, 4$

Thus centres are (6, 0) and (4, 0).



12. (b, c, d): We have $770 = (2)(5)(7)(11)$. We can assign each of 2, 5, 7, 11 in 4 ways. Thus, required number of solutions is $4^4 = 256 = 16^2 = 2^8$

13. (a, c): $a^2 = b^2 + c^2 - 2bc\cos A = b^2 + 8 - 4b$

$a = \sqrt{6} \Rightarrow b^2 - 4b + 2 = 0 \quad \therefore \quad b = 2 \pm \sqrt{2}, \Delta = \frac{1}{2}bc\sin A$

$\therefore \quad \Delta = \frac{1}{2}(2 \pm \sqrt{2})\sqrt{8} \times \frac{1}{\sqrt{2}} = 2 \pm \sqrt{2}$

14. (a, b, c, d): $\sin^{-1}|\sin x| = 0$ or 1

$\sin^{-1}|\sin x| = 0 \Rightarrow |\sin x| = \sin 0 \Rightarrow x = n\pi$

$\sin^{-1}|\sin x| = 1 \Rightarrow |\sin x| = \sin 1 \Rightarrow \sin x = \pm \sin 1$

If $\sin x = \sin 1 \Rightarrow x = 2n\pi + 1$ or $2n\pi + \pi - 1$

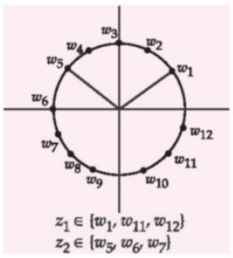
If $\sin x = -\sin 1 \Rightarrow x = 2n\pi - 1$ or $2n\pi + \pi + 1$

15. (c, d) : $w = \frac{\sqrt{3}+i}{2} = \frac{\sqrt{3}}{2} + i\frac{1}{2} = e^{i\pi/6}$

$w^n = e^{\frac{in\pi}{6}}$

For z_1, we have $\cos\frac{n\pi}{6} > \frac{1}{2}$ and

For z_2, we have $\cos\frac{n\pi}{6} < -\frac{1}{2}$



$z_1 = \frac{\sqrt{3}+i}{2}, \frac{\sqrt{3}-i}{2}, 1$

$z_2 = -1, \frac{-\sqrt{3}+i}{2}, \frac{-\sqrt{3}-i}{2}$

$\therefore \quad \angle z_1 O z_2 = \frac{5\pi}{6}, \frac{2\pi}{3}$

16. (b) : $u_{n+1} - u_n = 2(u_n - u_{n-1})$

$= 2^2(u_{n-1} - u_{n-2}) = 2^3(u_{n-2} - u_{n-3})$

.....

$= 2^{n-1}(u_2 - u_1) = 2^n$

$\Rightarrow \quad u_{n+1} = u_n + 2^n = u_{n-1} + 2^{n-1} + 2^n$

$= u_{n-2} + 2^{n-2} + 2^{n-1} + 2^n$

.....

$= u_1 + 2 + \ldots\ldots + 2^{n-1} + 2^n = 2^{n+1} - 1.$

17. (c) : Let $t_r = \frac{1}{r(r+1)(r+2)}$

$\Rightarrow \quad rt_r = \frac{1}{(r+1)(r+2)} = (r+3)t_{r+1}$

$\Rightarrow \quad rt_r - (r+1)t_{r+1} = nt_{r+1} \Rightarrow 2\sum_{r=1}^{n-1} t_{r+1} = t_1 - nt_n$

$\Rightarrow \quad 2\sum_{r=1}^{n} t_r = \frac{1}{2} - \frac{1}{(n+1)(n+2)}$

$\Rightarrow \quad 2\sum_{r=1}^{\infty} t_r = \frac{1}{2} - 0 = \frac{1}{2}.$

18. (b) : Let $t_r = r(r+1)(r+2)$

$\Rightarrow \quad (r+3)t_r - r\,t_{r+1} = 0 \Rightarrow 4t_r = rt_{r+1} - (r-1)t_r$

$\Rightarrow \quad 4\sum_{r=1}^{n} t_r = nt_{n+1} = 4!({}^{n+3}C_4).$

19. (b) : Given, $(1 + x + x^2)^n = \sum_{r=0}^{2n} a_r x^r$ ...(i)

Putting $x = 1$ in (i), we get

$a_0 + a_1 + a_2 + \ldots + a_{2n} = (1 + 1 + 1)^n = 3^n$

But $a_r = a_{2n-r}$ for $0 \le r < n - 1$.

$\therefore \quad 2(a_0 + a_1 + \ldots + a_{n-1}) + a_n = 3^n$

20. (a) : Replacing x by $-1/x$ in (i), we get

$\left(1 - \frac{1}{x} + \frac{1}{x^2}\right)^n = \sum_{r=0}^{2n} (-1)^r \frac{a_r}{x^r}$

Now, $a_0^2 - a_1^2 + a_2^2 - a_3^2 + \ldots + a_{2n}^2$ = coefficient of the constant term in $(a_0 + a_1 x + \ldots + a_{2n}x^{2n})$

$\left[a_0 - \frac{a_1}{x} + \frac{a_2}{x^2} - \frac{a_3}{x^3} + \ldots + \frac{a_{2n}}{x^{2n}}\right]$

= coefficient of the constant term in

$(1 + x + x^2)^n \left(1 - \frac{1}{x} + \frac{1}{x^2}\right)^n$

= coefficient of the constant term in

$\frac{(1+x+x^2)^n (x^2 - x + 1)^n}{x^{2n}}$

= coefficient of x^{2n} in $[(x^2 + 1)^2 - x^2]^n$

= coefficient of x^{2n} in $(1 + x^2 + x^4)^n$

= coefficient of y^n in $(1 + y + y^2)^n = a_n$

21. (b) : $\sum_{r=0}^{n}(-1)^r a_r {}^nC_r$ = coefficient of the constant term in $[a_0 + a_1x + a_2x^2 + ... + a_{2n}x^{2n}]$

$$\times\left[{}^nC_0 - {}^nC_1\left(\frac{1}{x}\right) + {}^nC_2\left(\frac{1}{x}\right)^2 + ... + (-1)^n\,{}^nC_n\left(\frac{1}{x}\right)^n\right]$$

= coefficient of the constant term in $(1+x+x^2)^n\left(1-\frac{1}{x}\right)^n$

= coefficient of x^n in $(x^2 + x + 1)^n (x - 1)^n$

= coefficient of x^n in $(x^3 - 1)^n$

= 0 as n is not a multiple of 3.

22. (b) : Since, $y^2 + 10y - 4x + 17 = 0$

$\Rightarrow$ $(y + 5)^2 - 25 - 4x + 17 = 0$

$\Rightarrow$ $(y + 5)^2 = 4x + 8$ $\Rightarrow$ $(y + 5)^2 = 4(x + 2)$

Let $y + 5 = Y$ and $x + 2 = X$, then $Y^2 = 4X$,

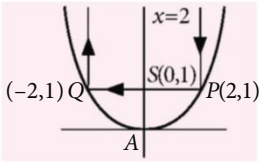
Focus is $X = 1$, $Y = 0$ *i.e.*, $(-1, -5)$

After reflection, the ray must pass through focus $(-1, -5)$.

23. (d) : Solving $x = 2$ and $x^2 = 4y$, we get $P(2, 1)$

Now, focus is $(0, 1)$

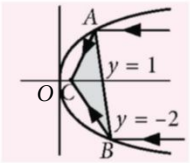
Equation of first reflection is $y = 1$ which is perpendicular to incident ray.



Hence, 2nd reflected ray is parallel to incident ray and pass through $(-2, 1)$ is $x = -2$ or $x + 2 = 0$

24. (a) : Solving $y = 1$ and $y^2 = x$, we get $A \equiv (1, 1)$

and solving $y = -2$ and $y^2 = x$, we get $B \equiv (4, -2)$



$\therefore$ After reflection both reflected rays pass through focus of the parabola $y^2 = x$ *i.e.*, $C \equiv \left(\frac{1}{4}, 0\right)$

$\therefore$ Required area $= \frac{1}{2}\begin{Vmatrix} 1 & 1 & 1 \\ 4 & -2 & 1 \\ \frac{1}{4} & 0 & 1 \end{Vmatrix} = \frac{21}{8}$ sq.units

25. (c) : Here $x = \frac{1}{1-\sin^2\theta} \Rightarrow \cos^2\theta = \frac{1}{x}$

and $y = \frac{1}{1-\cos^2\theta} \Rightarrow \sin^2\theta = \frac{1}{y}$

(A) $\sum_{n=0}^{\infty}\sin^{2n}\theta\cos^{2n}\theta = \frac{1}{1-\sin^2\theta\cos^2\theta}$

$= \frac{1}{1-\frac{1}{xy}} = \frac{xy}{xy-1}$

(B) $\sum_{n=0}^{\infty}\tan^{2n}\theta = \frac{1}{1-\tan^2\theta} = \frac{1}{1-\frac{x}{y}} = \frac{y}{y-x}$

(C) $\sum_{n=0}^{\infty}\sin^{2n}\theta\cos^{4n}\theta = \frac{1}{1-\sin^2\theta\cos^4\theta}$

$= \frac{1}{1-\frac{1}{x^2y}} = \frac{x^2y}{x^2y-1}$

(D) $\sum_{n=0}^{\infty}\cos^{2n}\theta\sin^{4n}\theta = \frac{1}{1-\cos^2\theta\sin^4\theta} = \frac{1}{1-\frac{1}{xy^2}}$

$= \frac{xy^2}{xy^2-1}$.

26. (b) : (A) Since function $f(x)$ is differentiable everywhere except at $x = 0$.

$\therefore$ $f(x)$ is not continuous at $x = 0$

Now, $f(x) = 2x^2 + \frac{2}{x^2} \Rightarrow f'(x) = 4x - \frac{4}{x^3}$

Critical points are $x = 1, -1$

Now, values of $f(x)$ at $x = -2, -1, 0, 1, 2$ are $\frac{17}{2}, 4, 1, 4, \frac{17}{2}$

$\Rightarrow$ $G = \frac{17}{2}, L = 1$

$\Rightarrow$ $[G + L] = 9$, $(G + L) = 10$

(B) Given $f(x) = x^3 - 6x^2 + 9x + 1$

$\therefore$ $f'(x) = 3x^2 - 12x + 9 = 3(x - 1)(x - 3)$

$\therefore$ $x = 1, 3$ but $3 \notin [0, 2]$

$G = \max\{f(0), f(1), f(2)\} = \max\{1, 5, 3\} = 5$

$L = \min\{f(0), f(1), f(2)\} = \min\{1, 5, 3\} = 1$

$\Rightarrow$ $[G + L] = 6$, $(G + L) = 6$

(C) Given $f(x) = \tan^{-1}x - \frac{1}{2}\ln x$

$\therefore$ $f'(x) = \frac{1}{1+x^2} - \frac{1}{2x} = -\frac{(x-1)^2}{2x(1+x^2)}$

$\therefore$ Critical points are $x = 0$ and $x = 1$

$\because$ At $x = 0$, $f'(x)$ does not exist

Then, $G = \max\left\{f\left(\frac{1}{\sqrt{3}}\right), f(1), f(\sqrt{3})\right\}$

$= \max\left\{\frac{\pi}{6} + \frac{1}{4}\ln 3, \frac{\pi}{4}, \frac{\pi}{3} - \frac{1}{4}\ln 3\right\} = \frac{\pi}{6} + \frac{1}{4}\ln 3$

and $L = \min\left\{f\left(\frac{1}{\sqrt{3}}\right), f(1), f(\sqrt{3})\right\}$

$= \min\left\{\frac{\pi}{6} + \frac{1}{4}\ln 3, \frac{\pi}{4}, \frac{\pi}{3} - \frac{1}{4}\ln 3\right\} = \frac{\pi}{3} - \frac{1}{4}\ln 3$

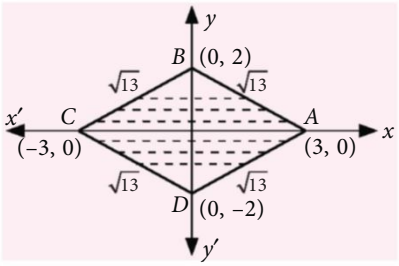
$\therefore$ $[G + L] = 1$, $(G + L) = 2$

27. (a) : (A) $\because d(x, y) = 2|x| + 3|y| = 6$ (given)

$\Rightarrow \frac{|x|}{3}+\frac{|y|}{2}=1$

$\therefore$ Perimeter, $\lambda = 4\sqrt{13}$

and area, $\mu = 4\times\frac{1}{2}\times 3\times 2 = 12$



Thus, $\frac{\lambda^2}{16}-\mu=1$ and $\lambda^2 - \mu^2 = 64$

(B) It is clear that orthocentre is (6, 6)

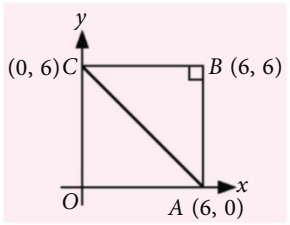
Circumcentre $\equiv$ (3, 3) and centroid $\equiv$ (4, 4)

$\therefore \lambda = \sqrt{(6-3)^2+(6-3)^2}$

$= \sqrt{9+9} = 3\sqrt{2}$

and $\mu = \sqrt{(4-3)^2+(4-3)^2}$

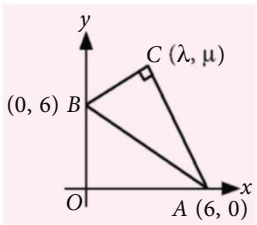
$= \sqrt{1+1} = \sqrt{2}$



$\therefore \lambda^2 - \mu^2 = 16$

(C) Slope of AC × Slope of $BC = -1$

$\Rightarrow \left(\frac{\mu-0}{\lambda-6}\right)\times\left(\frac{\mu-6}{\lambda-0}\right)=-1$



$\Rightarrow \mu^2 - 6\mu = -\lambda^2 + 6\lambda$

or $\lambda^2 + \mu^2 - 6\lambda - 6\mu = 0$

28. (5) : The pair of lines is

$3x^2 - xy + 3y^2 + (2x - 3y)\frac{(2x+3y)}{k} - 4\frac{(2x+3y)^2}{k^2} = 0$

Now, coefficient of x^2 + coefficient of $y^2 = 0$

$\Rightarrow 3+3+\frac{4-9}{k}-\frac{4(4+9)}{k^2}=0 \Rightarrow 6k-\frac{52}{k}=5.$

29. (51) : $f(x) = 1 - x^3 - x^4 - 2x^5 = (1 - x^4) - x^3 - 2x^5$

$= g(x) + h(x)$

$\therefore h(x) = -x^3 - 2x^5 \quad \therefore h(-5) = 6375$

30. (1) :

	Box I	Box II
Black marbles	a	c
White marbles	b	d

Given, $a + b + c + d = 25$

and $\frac{a}{a+b}\cdot\frac{c}{c+d}=\frac{27}{50}$

$\therefore 27(a + b)(c + d) = 50\,ac$, ac is divisible by 27.

$ac = 27\,k \Rightarrow (a + b)(c + d) = 50\,k$, $(a + b)(25 - (a + b)) = 50\,k$

$k = 3 \Rightarrow a + b = 15, c + d = 10, a = 9, b = 6, c = 9, d = 1$

$k = 2 \Rightarrow a + b = 20, c + d = 5, a = 18, b = 2, c = 3, d = 2$

In either case, the probability of two white marbles is

$\frac{bd}{(a+b)(c+d)}=\frac{6}{150}=\frac{4}{100}=\frac{1}{25}=\frac{m}{n}$

$\therefore \quad m = 1.$

31. (10) : $\lim_{x\to\pi/4}\frac{8-\sqrt{2}(\cos x+\sin x)^5}{1-\sin 2x}$

$=\sqrt{2}\lim_{x\to\pi/4}\frac{4\sqrt{2}-(\cos x+\sin x)^5}{1-\sin 2x}$

$=\sqrt{2}\lim_{x\to\pi/4}\frac{2^{5/2}-((\sin x+\cos x)^2)^{5/2}}{1-\sin 2x}$

$=\sqrt{2}\lim_{x\to\pi/4}\frac{2^{5/2}-(1+\sin 2x)^{5/2}}{2-(1+\sin 2x)}$

$=\sqrt{2}\lim_{u\to 2}\frac{2^{5/2}-u^{5/2}}{2-u}=\sqrt{2}\frac{5}{2}.2^{3/2} \quad (\because u = 1 + \sin 2x)$

$= 10$

32. (7) : $\frac{dy}{dx}+\frac{y}{x}=x^2y^6 \Rightarrow \frac{1}{y^6}\frac{dy}{dx}+\frac{1}{xy^5}=x^2$

Put $\frac{1}{y^5}=z \Rightarrow \frac{-5}{y^6}\frac{dy}{dx}=\frac{dz}{dx}$

$\therefore \quad -\frac{1}{5}\left(\frac{dz}{dx}\right)+\frac{z}{x}=x^2 \Rightarrow \text{I.F.}=\frac{1}{x^5}$

Hence solution is $\frac{1}{x^5y^5}=\frac{5}{2x^2}+c$

33. (6) : $F'(x) = 2(x + 3)(5 - x - 3) - 2x\,(5 - x)$

$= 12(1 - x)$

$F'(x) = 0 \quad \Rightarrow \quad x = 1$

$\therefore \quad M=\int_1^4 2t(5-t)\,dt = 33.$

34. (3) : We have $y = f(x) = x^{2/3} - x^{4/3}$

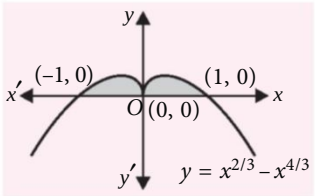
It meets the x-axis at $x = 0, \pm 1$.

Also, function is even.

$\therefore$ Required area,

$A=2\left[\int_0^1 (x^{2/3}-x^{4/3})\,dx\right]$

$=2\left[\frac{3x^{5/3}}{5}-\frac{3x^{7/3}}{7}\right]_0^1=\frac{12}{35}$



35. (5) : We have,

$1 + 2 + 3 + \ldots + (n - 2) \le 1224 \le 3 + 4 + \ldots + n$

$\Rightarrow \frac{(n-2)(n-1)}{2}\le 1224 \le \frac{(n-2)}{2}(n+3)$

Thus $n^2 - 3n - 2446 \le 0$ and $n^2 + n - 2454 \ge 0$

we get $49 < n < 51 \quad \therefore \quad n = 50$

Now, $\frac{n(n+1)}{2}-(2k+1)=1224 \quad \therefore \quad k=25$

Exam Between
6th to 20th April 2019

MOCK DRILL FOR JEE

1. If $21(x^2+y^2+z^2)=(x+2y+4z)^2$, then x, y, z are in
(a) A.P. (b) G.P.
(c) H.P. (d) none of these

2. Let $f(x)=[9^x-3^x+1]\ \forall\ x\in(-\infty, 1)$, then range of $f(x)$ is ([.] denotes the greatest integer function)
(a) {0, 1, 2, 3, 4, 5, 6} (b) {0, 1, 2, 3, 4, 5, 6, 7}
(c) {1, 2, 3, 4, 5, 6} (d) {1, 2, 3, 4, 5, 6, 7}

3. If f is an even function defined on the interval $(-5, 5)$, then the real values of x satisfying the condition $f(x)=f\left(\frac{x+1}{x+2}\right)$ are
(a) $\frac{-3\pm\sqrt{5}}{2}, \frac{3\pm\sqrt{5}}{2}$ (b) $\frac{-2\pm\sqrt{3}}{2}, \frac{-3\pm\sqrt{5}}{2}$
(c) $\frac{2\pm\sqrt{5}}{2}, \frac{-3\pm\sqrt{5}}{2}$ (d) none of these

4. The value of $\lim\limits_{x\to-\infty}\left\{\frac{x^4\sin\left(\frac{1}{x}\right)+x^2}{1+|x|^3}\right\}$ is
(a) 1 (b) −1 (c) −2 (d) 2

5. If $A(z_1)$, $B(z_2)$ and $C(z_3)$ are three points in the argand plane, where $|z_1+z_2|=||z_1|-|z_2||$ and $|(1-i)z_1+iz_3|=|z_1|+|z_3-z_1|$, where $i=\sqrt{-1}$, then
(a) A, B and C lie on a fixed circle with centre $\frac{(z_2+z_3)}{2}$.
(b) A, B and C are collinear points.
(c) ABC form an equilateral triangle.
(d) ABC form an obtuse angle triangle.

6. $\lim\limits_{x\to0}\left(\frac{1^x+2^x+3^x+.....+n^x}{n}\right)^{\frac{a}{x}}$ equals
(a) $\sqrt[a]{(n!)^n}$ (b) $\sqrt[n]{(n!)^a}$
(c) $(n!)^{an}$ (d) None of these

7. If $f(x)$ and $g(x)$ are periodic functions with periods 7 and 11 respectively, then the period of $F(x)=f(x)g\left(\frac{x}{5}\right)-g(x)f\left(\frac{x}{3}\right)$ is
(a) 177 (b) 222 (c) 433 (d) 1155

8. The solution of the system of equations $x+y=\frac{\pi}{2}$ and $\sin x+\sin y=\sqrt{2}$ is (x, y), then
(a) $x=2k\pi+\frac{\pi}{4}, y=\frac{\pi}{4}-2k\pi$
(b) $x=2k\pi-\frac{\pi}{2}, y=\frac{\pi}{2}+x$
(c) $x=2k\pi+\frac{\pi}{3}, y=\frac{\pi}{3}-2k\pi$
(d) $x=k\pi+\frac{\pi}{4}, y=\frac{\pi}{4}-k\pi\ (\forall\ k\in I)$

9. How many different nine digit numbers can be formed from the number 22 33 55 888 by rearranging its digits, so that the odd digits occupy even positions?
(a) 16 (b) 36 (c) 60 (d) 180

10. The solution set of the equation $\log_x 2\ \log_{2x}2=\log_{4x}2$ is
(a) $\{2^{-\sqrt{2}}, 2^{\sqrt{2}}\}$ (b) $\left\{\frac{1}{2}, 2\right\}$
(c) $\left\{\frac{1}{4}, 2^2\right\}$ (d) none of these

11. A straight line L with negative slope passes through the point (8, 2) and cuts the positive coordinate axes at points P and Q. As L varies, the absolute minimum value of $OP+OQ$ is (O is origin)
(a) 10 (b) 18 (c) 16 (d) 12

12. Domain of $\sin^{-1}[\sec x]$ ([.] is greatest integer less than or equal to x) is

(a) $\{(2n+1)\pi, (2n+9)\pi\} \cup \{(2m-1)\pi, 2m\pi + \pi/3), m \in I\}$

(b) $\{2n\pi, n \in I\} \cup \{[2m\pi, (2m+1)\pi, m \in I\}$

(c) $\{(2n+1)\pi, n \in I\} \cup \{[2m\pi, 2m\pi + \pi/3], m \in I\}$

(d) none of these

13. A factory is operating in two shifts, day and night, with 70 and 30 workers respectively. If per day mean wage of the day shift workers is ₹ 54 and per day mean wage of all the workers is ₹ 60, then per day mean wage of the night shift workers (in ₹) is

(a) 66 (b) 69 (c) 74 (d) 75

14. The coefficient of x^{49} in the product $(x-1)(x-3) \ldots (x-99)$ is

(a) -99^2 (b) 1

(c) -2500 (d) none of these

15. Let $A = \begin{pmatrix} 0 & 2b & c \\ a & b & -c \\ a & -b & c \end{pmatrix}$ be an orthogonal matrix then the values of a, b, c are

(a) $a = \pm\frac{1}{\sqrt{6}}, b = \pm\frac{1}{\sqrt{3}}, c = \pm\frac{1}{\sqrt{2}}$

(b) $a = \pm\frac{1}{\sqrt{2}}, b = \pm\frac{1}{\sqrt{3}}, c = \pm\frac{1}{\sqrt{6}}$

(c) $a = \pm\frac{1}{\sqrt{2}}, b = \pm\frac{1}{\sqrt{6}}, c = \pm\frac{1}{\sqrt{3}}$

(d) none of these

16. Let $A(-3, 2), B(2, -3)$ be the vertices of ΔABC. If the centroid of this triangle moves on the line $3x + 2y = 5$, then locus of the vertex C is the line

(a) $3x + 2y = 15$ (b) $3x + 2y = 20$

(b) $2x + 3y = 20$ (d) $3x - 2y = 15$

17. If $(1 + x + 2x^2)^{20} = a_0 + a_1x + a_2x^2 + \ldots + a_{40}x^{40}$, then $a_1 + a_3 + a_5 + \ldots + a_{37}$ equals to

(a) $2^{19}(2^{20} - 21)$ (b) $2^{20}(2^{19} - 19)$

(c) $2^{19}(2^{20} + 21)$ (d) none of these

18. If x, y, z are in A.P. and $\tan^{-1}x$, $\tan^{-1}y$ and $\tan^{-1}z$ are also in A.P., then

(a) $2x = 3y = 6z$ (b) $6x = 3y = 2z$

(c) $6x = 4y = 3z$ (d) $x = y = z$

19. Let $\Delta(x) = \begin{vmatrix} (x-2) & (x-1)^2 & x^3 \\ (x-1) & x^2 & (x+1)^3 \\ x & (x+1)^2 & (x+2)^3 \end{vmatrix}$, then the coefficient of x in $\Delta(x)$ is

(a) -3 (b) -2 (c) -1 (d) 0

20. The probability that out of 5 person, all having birthday in March, at least two have the same birthday, is

(a) $\frac{{}^{31}C_5}{(31)^5}$ (b) $\frac{(31)^5 - {}^{31}C_5}{(31)^5}$

(c) $\frac{(31)^3 \times {}^{31}C_2}{(31)^5}$ (d) none of these

21. $\int \frac{2\sin x}{(3+\sin 2x)}dx$ is equal to

(a) $\frac{1}{2}\ln\left|\frac{2+\sin x-\cos x}{2-\sin x+\cos x}\right| - \frac{1}{2\sqrt{2}}\tan^{-1}\left(\frac{\sin x+\cos x}{\sqrt{2}}\right) + c$

(b) $\frac{1}{2}\ln\left|\frac{2+\sin x-\cos x}{2-\sin x+\cos x}\right| - \frac{1}{2\sqrt{2}}\tan^{-1}\left(\frac{\sin x+\cos x}{\sqrt{2}}\right) + c$

(c) $\frac{1}{4}\ln\left|\frac{2+\sin x-\cos x}{2-\sin x+\cos x}\right| - \frac{1}{\sqrt{2}}\tan^{-1}\left(\frac{\sin x+\cos x}{\sqrt{2}}\right) + c$

(d) none of these

22. If the function $f(x) = \left[\frac{(x-5)^3}{A}\right]\sin(x-5) + a\cos(x-2)$, where [.] denotes the greatest integer function, is continuous and differentiable in [7, 9], then

(a) $A \in [8, 64]$ (b) $A \in (0, 8]$

(c) $A \in [64, \infty)$ (d) none of these

23. For any real number x, let $[x]$ denote the largest integer less than or equal to x. Let f be a real-valued function defined on the interval $[-10, 10]$ by

$$f(x) = \begin{cases} x - [x], & \text{if } [x] \text{ is odd} \\ 1 + [x] - x, & \text{if } [x] \text{ is even} \end{cases}$$

Then the value of $\frac{\pi^2}{10}\int_{-10}^{10} f(x)\cos \pi x\, dx$ is

(a) 4 (b) 5 (c) 3 (d) 2

24. Find the equation of the plane which passes through the z-axis and is perpendicular to the line $\frac{x-a}{\cos\theta} = \frac{y+2}{\sin\theta} = \frac{z-3}{0}$.

(a) $x + y\tan\theta = 0$ (b) $y + x\tan\theta = 0$

(c) $x\cos\theta - y\sin\theta = 0$ (d) $x\sin\theta - y\cos\theta = 0$

25. The coefficients of $x^2y^5z^3$ in the expansion of $(2x + y + 3z)^{10}$ is $k \cdot 10! \times 3^3$, then k equals

(a) $\frac{10!}{2!5!3!}2^2 3^2$ (b) $\frac{10!}{5!3!}3^3 2^2$

(c) $\frac{1}{3 \cdot 5!}$ (d) $\frac{10!}{2!5!3!}2^2 3^3$

26. A normal at $P(x, y)$ on a curve meets the x-axis at Q and N is the foot of the ordinate at P. If $NQ = \frac{x(1+y^2)}{1+x^2}$, then the equation of curve given that it passes through the point (3, 1) is

(a) $x^2 - y^2 = 8$ (b) $x^2 + 2y^2 = 11$

(c) $x^2 - 5y^2 = 4$ (d) none of these

27. The maximum value of $\frac{a\cos A + b\cos B + c\cos C}{a+b+c}$, where a, b, c and A, B, C have their usual meaning, is

(a) $\sin A$

(b) $4\sin(A/2)\sin(B/2)\sin(C/2)$

(c) $\sin(C/2)$ (d) none of these

28. The standard deviation of a distribution is 30 and each item is raised by 3, then new S.D. is

(a) 32 (b) 28 (c) 27 (d) 30

29. If the curves $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ and $\frac{x^2}{\alpha^2}-\frac{y^2}{\beta^2}=1$ cut each other orthogonally, then

(a) $a^2 + b^2 = \alpha^2 + \beta^2$ (b) $a^2 - b^2 = \alpha^2 - \beta^2$

(c) $a^2 - b^2 = \alpha^2 + \beta^2$ (d) $a^2 + b^2 = \alpha^2 - \beta^2$

30. If $x^3 = \int_0^z \frac{1}{\sqrt{1+6t^2}}dt$ then $\frac{1}{\frac{dx}{dz}}$ equals when $x = 1$, $z = 2$

(a) -15 (b) 15

(c) $\frac{1}{15}$ (d) $-\frac{1}{15}$

31. In [0, 1], Lagranges mean value theorem is NOT applicable to

(a) $f(x)=\begin{cases}\frac{1}{2}-x, & x<\frac{1}{2}\\ \left(\frac{1}{2}-x\right)^2, & x\geq\frac{1}{2}\end{cases}$

(b) $f(x)=\begin{cases}\frac{\sin x}{x}, x\neq 0\\ 1, \quad x=0\end{cases}$

(c) $f(x) = x|x|$ (d) $f(x) = |x|$

32. If $\vec{r}=x_1(\vec{a}\times\vec{b})+x_2(\vec{b}\times\vec{c})+x_3(\vec{c}\times\vec{a})$ and $4[\vec{a}\,\vec{b}\,\vec{c}]=1$, then $x_1 + x_2 + x_3$ is equal to

(a) $\frac{1}{2}\vec{r}\cdot(\vec{a}+\vec{b}+\vec{c})$ (b) $\frac{1}{4}\vec{r}\cdot(\vec{a}+\vec{b}+\vec{c})$

(c) $2\vec{r}\cdot(\vec{a}+\vec{b}+\vec{c})$ (d) $4\vec{r}\cdot(\vec{a}+\vec{b}+\vec{c})$

33. A bag contains n yellow balls and 3 black balls. Balls are drawn one by one without replacement till the black balls are drawn. The probability that this procedure for drawing ball will come to an end at the r^{th} draw

(a) $\frac{{}^nC_{r-3}}{{}^{n+3}C_r}$ (b) $\frac{{}^3C_1\times{}^nC_r}{n-r+4}$

(c) $\frac{3(r-1)r-2)}{(n+1)(n+2)(n+3)}$ (d) $\frac{3r(r-1)(r-2)}{(n+1)(n+2)(n+3)}$

34. Let $f: R \to R$ be a function such that

$f\left(\frac{x+y}{3}\right)=\frac{f(x)+f(y)}{3}$, $f(0) = 3, f'(0) = 3$, then

(a) $f(x)$ is continuous but not differentiable in R.

(b) $f(x)$ is differentiable but not continuous in R.

(c) $f(x)$ is both continuous and differentiable in R.

(d) $\frac{f(x)}{x}$ is differentiable in R.

35. If $\vec{a}$ and $\vec{b}$ are two unit vectors inclined at $\pi/3$, then $16[\vec{a}\,\vec{b}+\vec{a}\times\vec{b}\,\vec{b}]$ is

(a) 0 (b) 1

(c) -12 (d) 16

SOLUTIONS

1. (b) : $\because 21 = 1^2 + 2^2 + 4^2$

Now, $(1^2 + 2^2 + 4^2)(x^2 + y^2 + z^2) = (x + 2y + 4z)^2$

$\Rightarrow (1^2 + 2^2 + 4^2)(x^2 + y^2 + z^2) - (1\cdot x + 2\cdot y + 4\cdot z)^2 = 0$

$\Rightarrow \begin{vmatrix}1 & 2\\ x & y\end{vmatrix}^2 + \begin{vmatrix}2 & 4\\ y & z\end{vmatrix}^2 + \begin{vmatrix}4 & 1\\ z & x\end{vmatrix}^2 = 0$

$\Rightarrow (y - 2x)^2 + (2z - 4y)^2 + (4x - z)^2 = 0$

which is possible only when

$y - 2x = 0$, $2z - 4y = 0$ and $4x - z = 0$

or $\frac{y}{x}=2, \frac{z}{y}=2, \frac{z}{x}=4$

$\Rightarrow \frac{y}{x}=\frac{z}{y}\Rightarrow y^2 = zx$

$\therefore$ x, y, z are in GP.

2. (c) : $\because 9^x - 3^x + 1 = \left(3^x - \frac{1}{2}\right)^2 - \frac{1}{4} + 1$

$= \left(3^x - \frac{1}{2}\right)^2 + \frac{3}{4} \geq \frac{3}{4}$

$\therefore$ For $x \in (-\infty, 1)$, $\frac{3}{4} \leq 9^x - 3^x + 1 < \frac{28}{4}$

Now, $[9^x - 3^x + 1] = \{1, 2, 3, 4, 5, 6\}$

$\therefore$ $R_f = \{1, 2, 3, 4, 5, 6\}$

3. (a) : Given, $f(x) = f\left(\frac{x+1}{x+2}\right)$

$\therefore \quad f(-x) = f\left(\frac{-x+1}{-x+2}\right) = f(x)$

($\because f(x)$ is an even function)

$\Rightarrow \quad x = \frac{-x+1}{-x+2}$ and $-x = \frac{x+1}{x+2}$

$\Rightarrow \quad x^2 - 3x + 1 = 0$ and $x^2 + 3x + 1 = 0$

$\Rightarrow \quad \left(x-\frac{3}{2}\right)^2 = -1 + \frac{9}{4}$ and $\left(x+\frac{3}{2}\right)^2 = -1 + \frac{9}{4}$

$\Rightarrow \quad \left(x-\frac{3}{2}\right)^2 = \left(\frac{\sqrt{5}}{2}\right)^2$ and $\left(x+\frac{3}{2}\right)^2 = \left(\frac{\sqrt{5}}{2}\right)^2$

$\Rightarrow \quad x = \frac{3\pm\sqrt{5}}{2}$ and $x = \frac{-3\pm\sqrt{5}}{2}$

4. (b) : $\because x \to -\infty \quad \therefore \quad |x| = -x$

Then $\lim\limits_{x\to-\infty}\left\{\frac{x^4\sin\left(\frac{1}{x}\right)+x^2}{1-x^3}\right\}$ ... (i)

Put $x = -\frac{1}{t}$ in (i), we get

$$\lim_{t\to0}\left\{\frac{-\frac{\sin t}{t^4}+\frac{1}{t^2}}{1+\frac{1}{t^3}}\right\} = \lim_{t\to0}\left\{\frac{-\frac{\sin t}{t}+t}{t^3+1}\right\} = \frac{-1+0}{0+1} = -1$$

5. (a) : $\because |z_1 + z_2| = ||z_1| - |z_2||$

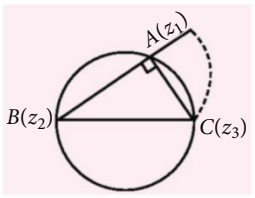
$\Rightarrow \quad \arg\left(\frac{z_1}{z_2}\right) = \pm\pi$

and $|z_1 + i(z_3 - z_1)| = |z_1| + |z_3 - z_1|$

$\Rightarrow \quad \arg\left(\frac{z_1}{z_3 - z_1}\right) = \frac{\pi}{2}$

$\therefore \quad$ Centre of circle is $\left(\frac{z_2+z_3}{2}\right)$

and ΔABC is right angled triangle $\angle A = 90°$



6. (a) : $\lim\limits_{x\to0}\left(\frac{1^x+2^x+3^x+\ldots+n^x}{n}\right)^{\frac{a}{x}}$ (1^∞ form)

$$= e^{\lim\limits_{x\to0}\left(\frac{1^x+2^x+\ldots+n^x}{n}-1\right)\left(\frac{a}{x}\right)}$$

$$= e^{\lim\limits_{x\to0}\left[\left(\frac{1^x-1}{x}\right)+\left(\frac{2^x-1}{x}\right)+\ldots+\left(\frac{n^x-1}{x}\right)\right]\frac{a}{n}}$$

$$= e^{\log(1\cdot2\ldots n)^{\frac{a}{n}}} = e^{\log(n!)^{\frac{a}{n}}} = (n!)^{\frac{a}{n}}$$

7. (d) : Given, period of $f(x)$ is 7

$\therefore \quad$ Period of $f\left(\frac{x}{3}\right)$ is $\frac{7}{1/3} = 21$

and period of $g(x)$ is 11

$\therefore \quad$ Period of $g\left(\frac{x}{5}\right)$ is $\frac{11}{1/5} = 55$

Now, T_1 = Period of $f(x)\, g\left(\frac{x}{5}\right) = 7\times55 = 385$

and T_2 = Period of $g(x) f\left(\frac{x}{3}\right) = 11\times21 = 231$

$\therefore \quad$ Period of $F(x)$ = L.C.M. of $\{T_1, T_2\}$
= L.C.M. of {385, 231}
= 7 × 11 × 3 × 5 = 1155

8. (a) : We have, $\sin x + \sin y = \sqrt{2}$

$\Rightarrow \quad 2\sin\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right) = \sqrt{2}$

$\Rightarrow \quad \cos\left(\frac{x-y}{2}\right) = 1 \qquad \left(\because x+y = \frac{\pi}{2}\right)$

$\Rightarrow \quad x - y = 2\cdot2\, k\pi = 4k\pi$

$\therefore \quad x = 2k\pi + \frac{\pi}{4}, y = \frac{\pi}{4} - 2k\pi, k \in I$

9. (c) : Required number of ways $= \frac{4!}{2!2!}\times\frac{5!}{2!3!} = 60$

10. (a) : $\log_x 2\log_{2x} 2 = \log_{4x} 2$

Here $x > 0$ and $x \neq 1, \frac{1}{2}, \frac{1}{4}$

$\Rightarrow \quad \frac{1}{\log_2 x}\cdot\frac{1}{\log_2 2x} = \frac{1}{\log_2 4x}$

$\Rightarrow \quad \log_2 x\cdot(1 + \log_2 x) = 2 + \log_2 x$

Put $\log_2 x = t$

$\Rightarrow \quad t = 2 \quad \Rightarrow \quad t = \pm\sqrt{2}$

$\Rightarrow \quad \log_2 x = \pm\sqrt{2} \quad \Rightarrow \quad x = 2^{\pm\sqrt{2}}$

$\therefore \quad x = 2^{-\sqrt{2}}, 2^{\sqrt{2}}$

11. (b) : The equation of the line L be $y - 2 = m(x - 8)$, $m < 0$. Coordinates of P and Q are $P\left(8-\frac{2}{m}, 0\right)$ and $Q(0, 2 - 8m)$.

Now, $OP + OQ = 8 - \frac{2}{m} + 2 - 8m = 10 + \frac{2}{(-m)} + 8(-m)$

$$\geq 10 + 2\sqrt{\frac{2}{(-m)}\times8(-m)} \geq 18$$

So, absolute minimum value of $OP + OQ = 18$

12. (c) : Here $f(x) = \sin^{-1}[\sec x]$

$f(x)$ is defined when $-1 \le [\sec x] \le 1$

$\because \quad [\sec x] \le 1 \quad \Rightarrow \quad \sec x < 2$

But $\sec x \ge 1 \quad \therefore \quad 1 \le \sec x < 2$

$$\Rightarrow \quad x \in \left[2m\pi, 2m\pi + \frac{\pi}{3}\right], m \in I \qquad ...(i)$$

And if $[\sec x] \ge -1$

$\therefore \quad \sec x \ge -1$

But $\sec x \ge 1$ and $\sec x \le -1$

$\therefore \quad \sec x = -1 \Rightarrow x = 2n\pi + \pi = (2n+1)\pi, n \in I \quad ...(ii)$

$\therefore$ From (i) and (ii), we get

$$D_f = \{(2n+1)\pi, n \in I\} \cup \left\{\left[2m\pi, 2m\pi + \frac{\pi}{3}\right], m \in I\right\}$$

13. (c) : Let average wage of night shift workers be x.

Now, $(70 \times 54) + (30 \times x) = 60 \times 100$

$\Rightarrow \quad 3780 + 30x = 6000 \Rightarrow 30x = 2220 \Rightarrow x = 74$

14. (c) : Here, number of factors = 50

$\therefore$ The coefficient of $x^{49} = -1 - 3 - 5 - ... - 99$

$$= -\frac{50}{2}(1+99) = -2500$$

15. (c) : $A = \begin{pmatrix} 0 & 2b & c \\ a & b & -c \\ a & -b & c \end{pmatrix}$ and $A^t = \begin{pmatrix} 0 & a & a \\ 2b & b & -b \\ c & -c & c \end{pmatrix}$

As A is orthogonal $\therefore AA^t = I$

$$\Rightarrow \begin{pmatrix} 0 & 2b & c \\ a & b & -c \\ a & -b & c \end{pmatrix}\begin{pmatrix} 0 & a & a \\ 2b & b & -b \\ c & -c & c \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$\Rightarrow \begin{pmatrix} 4b^2+c^2 & 2b^2-c^2 & -2b^2+c^2 \\ 2b^2-c^2 & a^2+b^2+c^2 & a^2-b^2-c^2 \\ -2b^2+c^2 & a^2-b^2-c^2 & a^2+b^2+c^2 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

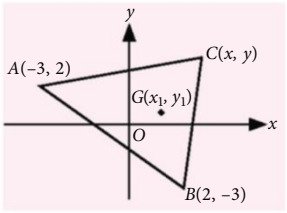
Using definition of equality of two matrices, we get,

$4b^2 + c^2 = 1$, $2b^2 - c^2 = 0$, $a^2 + b^2 + c^2 = 1$.

On solving them, we get

$$a = \pm\frac{1}{\sqrt{2}}, \; b = \pm\frac{1}{\sqrt{6}}, \; c = \pm\frac{1}{\sqrt{3}}$$

16. (b) : Let (x, y) be the co-ordinate of the vertex C and (x_1, y_1) be coordinate of centroid G.



$$\therefore \quad x_1 = \frac{x-3+2}{3}, \quad y_1 = \frac{y+2-3}{3}$$

$$\text{or} \quad (x_1, y_1) = \left(\frac{x-1}{3}, \frac{y-1}{3}\right)$$

Now (x_1, y_1) lies on $3x + 2y = 5$

$$\Rightarrow \quad 3x_1 + 2y_1 = 5$$

$$\Rightarrow \quad \frac{3(x-1)}{3} + \frac{2(y-1)}{3} = 5$$

$\Rightarrow \quad 3x + 2y - 3 - 2 = 15$

$\Rightarrow \quad 3x + 2y = 20$ which is required locus of vertex C.

17. (a) : $\because (1 + x + 2x^2)^{20} = a_0 + a_1x + a_2x^2 + a_3x^3 + a_4x^4 + a_5x^5 + ... + a_{37}x^{37} + a_{38}x^{38} + a_{39}x^{39} + a_{40}x^{40}$ ...(i)

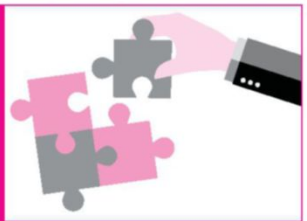
Putting $x = 1$ and $x = -1$ in (i) and subtracting, we get

$4^{20} - 2^{20} = 2(a_1 + a_3 + a_5 + .. + a_{37} + a_{39})$

$$\Rightarrow \quad a_1 + a_3 + a_5 + ... + a_{37} = 2^{19}(2^{20} - 1) - a_{39} \qquad ...(ii)$$

Now, $(1 + x + 2x^2)^{20} = \frac{20!}{\alpha!\beta!\gamma!}(1)^{\alpha}(x)^{\beta}(2x^2)^{\gamma}$

PUZZLE CORNER



MATHDOKU

Introducing MATHDOKU, a mixture of ken-ken, sudoku and Mathematics.

In this puzzle 6 × 6 grid is given, your objective is to fill the digits 1-6 so that each appear exactly once in each row and each column.

Notice that most boxes are part of a cluster. In the upper-left corner of each multibox cluster is a value that is multiple of its numbers. For example, if that value is 3 for a two-box cluster, you know that only 1 and 3 can go in there. But it is your job to determine which number goes where! A few cluster may have just one box and that is the number that fills that box.

6+		4	18+		
	17+		12×	3×	
1–					
	1–		24×		
12×		11+		3+	
5–			9+		

Readers can send their responses at editor@mtg.in or post us with complete address. Winners' name with their valuable feedback will be published in next issue.

For a_{39}, $\beta + 2\gamma = 39$ and $\alpha + \beta + \gamma = 20$

$\Rightarrow \quad \gamma = 19, \beta = 1, \alpha = 0$

$\therefore \quad a_{39} = \dfrac{20!}{0!1!19!} \cdot 2^{19} = 20 \cdot 2^{19}$

$\therefore$ From (ii), $a_1 + a_3 + a_5 + ... + a_{37} = 2^{19}(2^{20} - 21)$

18. (d) : $\because$ As x, y, z are in A.P. $\therefore \quad 2y = x + z$ ...(i)

$\tan^{-1}x$, $\tan^{-1}y$ and $\tan^{-1}z$ are in A.P., then

$2\tan^{-1}y = \tan^{-1}x + \tan^{-1}z$

$$2\tan^{-1} y = \tan^{-1}\left(\frac{x+z}{1-xz}\right)$$

$$\Rightarrow \quad \tan^{-1}\left(\frac{2y}{1-y^2}\right) = \tan^{-1}\left(\frac{x+z}{1-xz}\right)$$

Thus, $y^2 = xz$ ... (ii)

From (i) and (ii), we get $x = y = z$.

19. (b) : $\Delta(x) = \begin{vmatrix} (x-2) & (x-1)^2 & x^3 \\ (x-1) & x^2 & (x+1)^3 \\ x & (x+1)^2 & (x+2)^3 \end{vmatrix}$,

Applying $R_2 \to R_2 - R_1$ and $R_3 \to R_3 - R_1$, we get

$$\Delta(x) = \begin{vmatrix} (x-2) & (x-1)^2 & x^3 \\ 1 & (2x-1) & 3x^2+3x+1 \\ 2 & 4x & 6x^2+12x+8 \end{vmatrix}$$

Applying $R_3 \to R_3 - 2R_2$, we get

$$\Delta(x) = \begin{vmatrix} (x-2) & (x-1)^2 & x^3 \\ 1 & (2x-1) & 3x^2+3x+1 \\ 0 & 2 & 6x+6 \end{vmatrix}$$

Expanding along C_1, we get

$\Delta(x) = (x - 2)\{(2x - 1)(6x + 6) - 2(3x^2 + 3x + 1)\}$

$\quad -1\{(x - 1)^2(6x + 6) - 2x^3\}$

$\therefore$ Coefficient of x in $\Delta(x) = -8 + 6 = -2$

20. (b) : Number of days in month of March = 31

$\therefore \quad n(S) = (31)^5$

$n(A)$ = At least two have the same birthday

= $n(S)$ – Number of ways in which all candidates have the different birthday.

$= (31)^5 - {}^{31}C_5$

$$\therefore \quad P(A) = \frac{(31)^5 - {}^{31}C_5}{(31)^5}$$

21. (c) : Let $I = \int \dfrac{2\sin x}{(3+\sin 2x)}dx$

$$= \int \frac{\sin x + \cos x + \sin x - \cos x}{(3+\sin 2x)}dx$$

$$= \underbrace{\int \frac{\sin x + \cos x}{3+\sin 2x}dx}_{I_1} - \underbrace{\int \frac{-\sin x + \cos x}{(3+\sin 2x)}dx}_{I_2}$$

Putting $t_1 = \sin x - \cos x$ in I_1 and $t_2 = \sin x + \cos x$ in I_2, we get

$$I = \int \frac{dt_1}{[3+(1-t_1^2)]} - \int \frac{dt_2}{[3+(t_2^2-1)]} = \int \frac{dt_1}{4-t_1^2} - \int \frac{dt_2}{2+t_2^2}$$

$$= \frac{1}{4}\ln\left|\frac{2+t_1}{2-t_1}\right| - \frac{1}{\sqrt{2}}\tan^{-1}\left(\frac{t_2}{\sqrt{2}}\right) + C$$

$$= \frac{1}{4}\ln\left|\frac{2+\sin x - \cos x}{2-\sin x + \cos x}\right| - \frac{1}{\sqrt{2}}\tan^{-1}\left(\frac{\sin x + \cos x}{\sqrt{2}}\right) + C$$

22. (c) : $[x]$ is not continuous & differentiable at integral values (points). So, $f(x)$ is continuous and differentiable in [7, 9] if $\left[\dfrac{(x-5)^3}{A}\right] = 0$

$\Rightarrow \quad A \geq (9 - 5)^3 \Rightarrow A \geq 64 \therefore A \in [64, \infty)$

23. (a) : We have $f(x) = \begin{cases} x-1, & 1 \leq x < 2 \\ 1-x, & 0 \leq x < 1 \end{cases}$

$f(x)$ is periodic with period 2 and also it is an even function.

Also, $\cos \pi x$ has period 2

$$\therefore \quad I = \int_{-10}^{10} f(x)\cos \pi x\, dx$$

$$= 2\int_0^{10} f(x)\cos \pi x\, dx \qquad (\because f(x)\cos \pi x \text{ is an even function})$$

$$= 2 \times 5\int_0^2 f(x)\cos \pi x\, dx$$

$$= 10\left[\int_0^1 (1-x)\cos \pi x\, dx + \int_1^2 (x-1)\cos \pi x\, dx\right]$$

$$= 10(I_1 + I_2)$$

$$I_2 = \int_1^2 (x-1)\cos \pi x\, dx \qquad (\text{put } x - 1 = t)$$

$$I_2 = -\int_0^1 t\cos \pi t\, dt$$

$$I_1 = \int_0^1 (1-x)\cos \pi x\, dx = -\int_0^1 x\cos(\pi x)\, dx$$

$\therefore \quad I = 10\left[-2\int_0^1 x\cos \pi x\, dx\right] = -20\left[x\frac{\sin \pi x}{\pi} + \frac{\cos \pi x}{\pi^2}\right]_0^1$

$= -20\left[-\frac{1}{\pi^2} - \frac{1}{\pi^2}\right] = \frac{40}{\pi^2}$

$\therefore \quad \frac{\pi^2}{10} I = 4$

24. (a) : The plane is perpendicular to the line $\frac{x-a}{\cos\theta} = \frac{y+2}{\sin\theta} = \frac{z-3}{0}$.

Hence, the direction ratios of the normal of the plane are cos θ, sin θ and 0.

Now, the required plane passes through the z-axis. Hence, the point (0, 0, 0) lies on the plane.

$\therefore$ Equation of the plane is given by

$\cos\theta\,(x-0) + \sin\theta\,(y-0) + 0\,(z-0) = 0$

$\Rightarrow \quad x\cos\theta + y\sin\theta = 0 \Rightarrow x + y\tan\theta = 0$

25. (c) : $(2x + y + 3z)^{10}$

$= \sum_{x_1+x_2+x_3=10} \frac{10!}{x_1!x_2!x_3!}(2x)^{x_1}(y)^{x_2}(3z)^{x_3}$

General term in the expansion

$= \frac{10!}{x_1!x_2!x_3!}(2x)^{x_1}(y)^{x_2}(3z)^{x_3}$

$= \frac{10!}{x_1!x_2!x_3!}2^{x_1}3^{x_3}x^{x_1}\cdot y^{x_2}\cdot z^{x_3}$

Now, for the coefficient of $x^2y^5z^3$, we have $x_1 = 2$, $x_2 = 5$, $x_3 = 3$

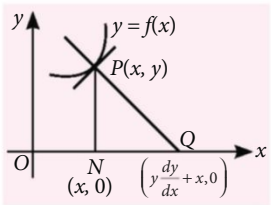
$\therefore$ Required coefficient of $x^2y^5z^3$ is $\frac{10!}{2!5!3!}2^2\cdot 3^3$

Now, $\frac{10!}{2!5!3!}2^2\,3^3 = k \times 10! \times 3^3$ (Given)

$\therefore \quad k = \frac{4}{2!3!5!} = \frac{4}{2\cdot 6\cdot 120} = \frac{1}{360} = \frac{1}{3\cdot 5!}$

26. (c) : Equation of normal at point $P(x, y)$ is

$Y - y = -\frac{dx}{dy}(X - x)$



Now, $NQ = y\frac{dy}{dx} = \frac{x(1+y^2)}{1+x^2}$

$\Rightarrow \quad \frac{xdx}{1+x^2} = \frac{ydy}{1+y^2}$

$\Rightarrow \quad \ln(1+x^2) = \ln(1+y^2) + \ln c$

$\Rightarrow \quad 1 + y^2 = \frac{1+x^2}{c}$

Since, it passes through (3, 1). Thus, $1 + 1 = \frac{1+(3)^2}{c}$ or $c = 5$

Required curve is $5 + 5y^2 = 1 + x^2$ or $x^2 - 5y^2 = 4$.

27. (b) : $\frac{a\cos A + b\cos B + c\cos C}{a+b+c}$

$= \frac{R(\sin 2A + \sin 2B + \sin 2C)}{2R(\sin A + \sin B + \sin C)}$

$\left[\because \frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} = 2R \text{ (sine formula)}\right]$

$= \frac{4\sin A\sin B\sin C}{2\left(4\cos\frac{A}{2}\cos\frac{B}{2}\cos\frac{C}{2}\right)} = 4\sin\frac{A}{2}\sin\frac{B}{2}\sin\frac{C}{2}$

28. (d) : S.D. of a series remains unaltered if each item is raised (reduced) by the same scalar quantity. S.D. is independent of change of origin.

29. (b) : The given curves are $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ ...(i)

and $\frac{x^2}{\alpha^2} - \frac{y^2}{\beta^2} = 1$ ...(ii)

Slope of (i) $= \frac{-b^2x}{a^2y} = m_1$ (say),

And slope of (ii) $= \frac{\beta^2x}{\alpha^2y} = m_2$ (say)

$\therefore \quad m_1m_2 = -1$

$\Rightarrow \quad b^2\beta^2x^2 = a^2\alpha^2y^2$ ...(iii)

Subtracting (ii) from (i), we get

$x^2\left(\frac{1}{a^2} - \frac{1}{\alpha^2}\right) = -y^2\left(\frac{1}{b^2} + \frac{1}{\beta^2}\right)$ ...(iv)

From (iii) and (iv), we get

$\frac{\frac{1}{a^2} - \frac{1}{\alpha^2}}{b^2\beta^2} = -\frac{\left(\frac{1}{b^2} + \frac{1}{\beta^2}\right)}{a^2\alpha^2}$

$\Rightarrow \quad \alpha^2 - \beta^2 = a^2 + b^2$

30. (b) : $x^3 = \int_0^z \frac{1}{\sqrt{1+6t^2}}dt$

$\Rightarrow \quad 3x^2\frac{dx}{dz} = 1\cdot\frac{1}{\sqrt{1+6z^2}}$

$\Rightarrow \quad \frac{dx}{dz} = \frac{1}{3x^2\sqrt{1+6z^2}} \Rightarrow \frac{dz}{dx} = 3x^2\sqrt{1+6z^2}$

$\therefore \quad \left(\frac{dz}{dx}\right)_{x=1,z=2} = 3\times 5 = 15$

31. (a) : Functions in options (b), (c) and (d) are continuous on [0, 1] and differentiable in (0, 1).

Now, for $f(x)=\begin{cases}\frac{1}{2}-x, & x<\frac{1}{2}\\ \left(\frac{1}{2}-x\right)^2, & x\geq\frac{1}{2}\end{cases}$

Here, $Lf'\left(\frac{1}{2}\right)=-1$ and $Rf'\left(\frac{1}{2}\right)=2\left(\frac{1}{2}-\frac{1}{2}\right)(-1)=0$

$\therefore \quad Lf'\left(\frac{1}{2}\right)\neq Rf'\left(\frac{1}{2}\right)$

$\Rightarrow$ f is non differentiable at $x=\frac{1}{2}\in(0,1)$

$\therefore$ LMVT is NOT applicable to $f(x)$ in [0, 1].

32. (d) : $\vec{r}=x_1(\vec{a}\times\vec{b})+x_2(\vec{b}\times\vec{c})+x_3(\vec{c}\times\vec{a})$

$\Rightarrow \quad \vec{r}.\vec{a} = x_2[\vec{a}\,\vec{b}\,\vec{c}], \vec{r}\cdot\vec{b}=x_3[\vec{b}\,\vec{c}\,\vec{a}]$

and $\vec{r}\cdot\vec{c}=x_1[\vec{c}\,\vec{a}\,\vec{b}] = x_1[\vec{a}\,\vec{b}\,\vec{c}]$

$\Rightarrow \quad x_1+x_2+x_3=4\vec{r}\cdot(\vec{a}+\vec{b}+\vec{c})$

33. (c) : If the procedure for drawing balls has to come to an end at the r^{th} draw then all balls except one black must be drawn in the first $(r-1)$ draws. Consider the following events

A = getting 2 black balls in first $(r-1)$ draws

B = getting a black ball in the r^{th} draw

$\therefore$ Required probability = $P(A\cap B)$

$$=P(A)P\left(\frac{B}{A}\right)=\frac{{}^3C_2\times{}^nC_{r-3}}{{}^{n+3}C_{r-1}}\times\frac{1}{n-(r-4)}$$

$$=\frac{3\times(r-1)(r-2)}{(n+1)(n+2)(n+3)} \qquad \text{(on simplification)}$$

34. (c) : Given, $f\left(\frac{x+y}{3}\right)=\frac{f(x)+f(y)}{3}$ …(i)

Replacing x by $3x$ & y by zero in (i), we get

$$f\left(\frac{3x+0}{3}\right)=\frac{f(3x)+f(0)}{3}$$

$\Rightarrow \quad f(3x)-3f(x)=-f(0)$ …(ii)

and $f'(x)=\lim\limits_{h\to0}\dfrac{f(x+h)-f(x)}{h}$

$$=\lim_{h\to0}\frac{f\left(\frac{3x+3h}{3}\right)-f(x)}{h}=\lim_{h\to0}\frac{\frac{f(3x)+f(3h)}{3}-f(x)}{h}$$

$$=\lim_{h\to0}\frac{f(3x)+f(3h)-3f(x)}{3h}$$

$$=\lim_{h\to0}\frac{f(3h)-f(0)}{3h} \qquad \text{(using (ii))}$$

$=f'(0)=3$ (given) $\Rightarrow \quad f(x)=3x+k$

$\Rightarrow \quad f(x)=3(x+1) \qquad (\because f(0)=3 \therefore k=3)$

which is a linear function which is continuous and differentiable $\forall\, x\in R$.

35. (c) : Given $\vec{a}$ and $\vec{b}$ are two unit vectors, *i.e.*, $|\vec{a}|=|\vec{b}|=1$ and angle between them is $\pi/3$.

$$\therefore \quad \sin\theta=\frac{|\vec{a}\times\vec{b}|}{|\vec{a}||\vec{b}|}\Rightarrow\sin\frac{\pi}{3}=|\vec{a}\times\vec{b}|$$

$$\Rightarrow \quad \frac{\sqrt{3}}{2}=|\vec{a}\times\vec{b}|$$

Now, $[\vec{a}\,\vec{b}+\vec{a}\times\vec{b}\,\vec{b}]=[\vec{a}\,\vec{b}\,\vec{b}]+[\vec{a}\,\vec{a}\times\vec{b}\,\vec{b}]=0+[\vec{a}\,\vec{a}\times\vec{b}\,\vec{b}]$

$=(\vec{a}\times\vec{b})\cdot(\vec{b}\times\vec{a})=-(\vec{a}\times\vec{b})\cdot(\vec{a}\times\vec{b})$

$=-|\vec{a}\times\vec{b}|^2=-\frac{3}{4}$

SAMURAI SUDOKU

SOLUTION · JANUARY 2019

4	6	7	1	9	3	8	2	5
5	8	3	7	6	2	1	9	4
1	2	9	4	8	5	7	6	3
8	1	2	6	5	7	4	3	9
7	9	5	3	4	1	6	8	2
3	4	6	8	2	9	5	1	7
9	3	1	5	7	6	2	4	8
2	5	4	9	1	8	3	7	6
6	7	8	2	3	4	9	5	1

3	1	2	5	4	7	8	9	6
6	9	5	1	2	8	4	3	7
4	7	8	3	6	9	1	5	2
7	2	1	9	8	3	5	6	4
8	3	4	6	7	5	9	2	1
9	5	6	4	1	2	7	8	3
5	6	7	2	9	4	3	1	8
1	8	9	7	3	6	2	4	5
2	4	3	8	5	1	6	7	9

			7	2	4	3	5	9
			1	8	5	4	6	7
			6	3	9	2	8	1

8	1	6						
3	9	2						
7	5	4						

7	4	6	3	8	9	5	1	2
2	9	3	5	4	1	8	6	7
5	1	8	2	7	6	4	9	3
6	2	1	4	5	8	3	7	9
4	3	9	6	2	7	1	8	5
8	7	5	1	9	3	2	4	6
9	5	7	8	3	4	6	2	1
1	8	2	7	6	5	9	3	4
3	6	4	9	1	2	7	5	8

6	7	8	2	3	4	9	5	1
2	5	4	9	1	8	3	7	6
9	3	1	5	7	6	2	4	8
3	4	6	8	2	9	5	1	7
7	9	5	3	4	1	6	8	2
8	1	2	6	5	7	4	3	9
5	8	3	7	6	2	1	9	4
1	2	9	4	8	5	7	6	3

1	6	2	3	9	4	7	5	8
7	9	4	6	8	5	1	2	3
8	5	3	2	1	7	6	9	4

Challenging PROBLEMS ON JEE Advanced

Single Option Correct Type

1. The sequence $< a_n >$ is defined by $a_1 = 0$ and $a_{n+1} = a_n + 4n + 3$, $n \geq 1$. Then the value of

$$\lim_{n\to\infty} \frac{\sqrt{a_n} + \sqrt{a_{4n}} + \sqrt{a_{4^2 n}} + ... + \sqrt{a_{4^{10} n}}}{\sqrt{a_n} + \sqrt{a_{2n}} + \sqrt{a_{2^2 n}} + ... + \sqrt{a_{2^{10} n}}} \text{ is}$$

(a) 600 (b) 683 (c) 800 (d) 863

2. The graph of the function $y = x^3 + ax + b$ has exactly three common points with the co-ordinate axes and they are vertices of a right triangle. a, b are real parameters then value of a is

(a) $\frac{-3}{\sqrt{2}}$ (b) $\frac{3}{\sqrt{2}}$ (c) $3\sqrt{2}$ (d) $2\sqrt{3}$

3. A right angled trapezium with area 10 and altitude 4 is divided into two circumscribed trapeziums by a line parallel to its bases, the ratio of their in-radii is

(a) 2 (b) 3 (c) $\frac{2}{3}$ (d) 6

4. The function $f(x) = \begin{cases} e^{-\frac{1}{|x|}}\left(\sqrt{2} + \sin\frac{1}{x}\right), & x \neq 0 \\ 0, & x = 0 \end{cases}$

has a global minimum value at $x =$

(a) 0 (b) 1 (c) –1 (d) nowhere

5. Evaluate : $\lim_{m\to\infty}\left(m^3 \int_m^{2m} \frac{x\,dx}{x^5+1}\right)$, $(m \in N, 0 < x < 1)$

(a) $\frac{22}{7}$ (b) $\frac{7}{22}$ (c) $\frac{24}{7}$ (d) $\frac{7}{24}$

6. In ΔABC, $\sqrt{3}\sin C = 2\sec A - \tan A$ and one of the sides is of length 2, then maximum area of the triangle ABC is

(a) $2\sqrt{3}$ sq. units (b) $4\sqrt{3}$ sq. units

(c) $\frac{\sqrt{3}}{2}$ sq. units (d) $\frac{\sqrt{3}}{4}$ sq. units

7. Let $a, b, c \in R$ such that $abc = p$ and $\frac{b}{a} = r$ where p and r are fixed positive numbers, then minimum distance of the point (a, b, c) from origin is $\sqrt{3}\left(\frac{p(r^2+1)}{\lambda}\right)^{\frac{1}{3}}$ where $\lambda =$

(a) $2p$ (b) p (c) $2r$ (d) r

8. If $A(1, 3, 4)$, $B(2, 4, -7)$, then point P on plane $x + y + 2z + 1 = 0$ such that $PA + PB$ is minimum is

(a) $\left(\frac{33}{20}, \frac{73}{20}, \frac{-63}{20}\right)$ (b) $\left(\frac{55}{18}, \frac{21}{18}, \frac{-87}{18}\right)$

(c) (0, 0, 0) (d) $\left(\frac{35}{12}, \frac{-61}{12}, \frac{-13}{12}\right)$

9. In ΔABC, $\tan A : \tan B : \tan C = 3 : 4 : 5$ then $\sin A \sin B \sin C =$

(a) $\frac{\sqrt{5}}{7}$ (b) $\frac{2\sqrt{5}}{7}$ (c) $\frac{\sqrt{7}}{5}$ (d) $\frac{2\sqrt{7}}{5}$

10. Eight boxes, each a unit cube, are packed in a $2 \times 2 \times 2$ crate, open at the top. The boxes are taken out one by one, the number of ways this can be done is

(a) 252 (b) 2520 (c) 1256 (d) 1250

11. The number of pairs (a, b) of real numbers such that $a + \log a = b$ and $b + \log b = a$ is/are

(a) 0 (b) 1 (c) 2 (d) 4

12. Consider the two functions, for $x > 0$,

$$f(x) = \frac{\left(1+\frac{1}{x}\right)^x}{(1+x)^{\frac{1}{x}}} \text{ and } g(x) = \left(1+\frac{1}{x}\right)^x - (1+x)^{\frac{1}{x}}, \text{ then}$$

(a) both f and g are increasing functions

(b) both f and g are decreasing functions

(c) f is increasing and g is a decreasing function

(d) f is decreasing and g is a increasing function

By : Tapas Kr. Yogi, Visakhapatnam Mob : 09533632105

13. If $A=\begin{bmatrix}-1 & 1\\ 0 & -2\end{bmatrix}=B^3+C^3$ where B and C are 2×2 matrices with integer elements then $Tr(B)+Tr(C)=$

(a) 1 (b) −1 (c) 3 (d) −3

Comprehension Type

Paragraph for Q. No. 14 and 15

The curve C has the equation $y=\dfrac{x}{\sqrt{x^2-2x+a}}$

14. If $a>1$ then C has

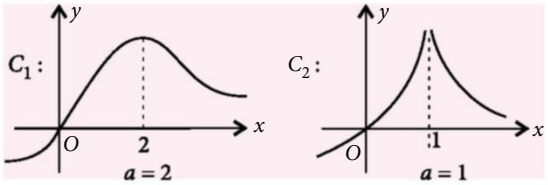
(a) exactly one stationary point

(b) exactly two stationary points

(c) no stationary point

(d) one or two stationary point depends on value of a.

15. Consider the two graphs C_1 and C_2 :



(a) Both C_1, C_2 are correct graphs

(b) Only C_1 is correct graph

(c) Only C_2 is correct graph

(d) Neither C_1 nor C_2 are correct graphs

Paragraph for Q. No. 16 and 17

The T-polynomials are defined by $T_0(x)=1$ and for $n\geq 1$, $\dfrac{dT_n}{dx}=n\cdot T_{n-1}(x)$ and $\int_0^1 T_n(x)dx=0$

16. $T_4(x)=$

(a) $x^2(x-1)^2+c$ (b) $x^2(x-1)^3+c$

(c) $x(x-1)^2+c$ (d) $x^3(x-1)^2+c$

17. For $n\geq 2$, $T_n(1)-T_n(0)=$

(a) 0 (b) 1 (c) −1 (d) $\dfrac{1}{2}$

Paragraph for Q. No. 18 and 19

Consider the set of points (x, y) in the plane which satisfy $x^2+y^2\leq 100$ and $\sin(x+y)\geq 0$.

18. Let A_1 and A_2 be areas of regions within $x^2+y^2\leq 100$ which satisfy $\sin(x+y)>0$ and $\sin(x+y)<0$, then

(a) $A_1>A_2$ (b) $A_1<A_2$

(c) $A_1=A_2$ (d) $A_1=2A_2$

19. The area of the region $x^2+y^2\leq 100$ and $\sin(x+y)\geq 0$ is

(a) 25π (b) 50π

(c) 100π (d) 200π

SOLUTIONS

1. (b) : Using the recurrence relation,

$a_k=a_{k-1}+4(k-1)+3=a_{k-2}+4(k-2)+4(k-1)+2\cdot 3$

$=a_1+4(1+2+3+....+k-1)+(k-1)\cdot 3$

$=(2k+3)(k-1)$

So, $\lim\limits_{n\to\infty}\dfrac{\sqrt{a_{kn}}}{n}=\lim\limits_{n\to\infty}\sqrt{\left(2k+\dfrac{3}{n}\right)\left(k-\dfrac{1}{n}\right)}=\sqrt{2}\,k$

So, required limit $=\dfrac{1+4+4^2+...+4^{10}}{1+2+2^2+...+2^{10}}=\dfrac{4^{11}-1}{3(2^{11}-1)}$

$=\dfrac{2^{11}+1}{3}=683$

2. (a) : The condition in the problem implies $x^3+ax+b=0$ has a double root α and a simple root β.

$\therefore\quad x^3+ax+b=(x-\alpha)^2(x-\beta)$

and $A(\alpha, 0)$, $B(\beta, 0)$ and $C(0, b)$ with $\angle ACB=90°$, $\alpha\beta<0$.

So, $OA\cdot OB=OC^2$ and $\alpha+\alpha+\beta=0$

Hence, $b=-\alpha^2\beta$ and $-\alpha\beta=b^2$ gives

$\alpha=\pm\dfrac{1}{\sqrt[4]{2}},\ \beta=\mp\sqrt[4]{8}$

So, $a=\alpha^2+2\alpha\beta=\dfrac{-3}{\sqrt{2}}$

3. (a) : We know that for given quadrilateral $ABCD$,

$AB+CD=AD+BC$

From diagram, $BQ=c+a-m_1$

and $CQ=b+c-m_2$

In ΔBQR, $BQ^2=QR^2+BR^2$

$\Rightarrow\quad (c+a-m_1)^2=m_1^2+(a-c)^2$

So, $m_1=\dfrac{2ac}{a+c}$.

Similarly, $m_2=\dfrac{2bc}{b+c}$

But, $\dfrac{m_1}{m_2}=\dfrac{a-c}{c-b}\ \Rightarrow\ c^2=ab$

Required ratio of in-radii

$=\dfrac{m_1/2}{m_2/2}=2$

4. (a) : We have,

$$f'(x)=\begin{cases}\dfrac{1}{x^2}e^{\frac{-1}{|x|}}\left(\left(\sqrt{2}+\sin\dfrac{1}{x}\right)\text{sgn}(x)-\cos\dfrac{1}{x}\right), & x\neq 0\\ 0, & x=0\end{cases}$$

Since, $\left|\sin\dfrac{1}{x}\pm\cos\dfrac{1}{x}\right|\leq\sqrt{2}$, we have

$f'(x) \geq 0$ if $x > 0$ and $f'(x) \leq 0$ if $x < 0$ and $f'(0) = 0$.
So, global minimum at $x = 0$, because $f(x) > f(0) = 0$ for $x \neq 0$.

5. (d) : Using, Sandwich rule,

$$m^3 \int_m^{2m} \frac{dx}{(x+1)^4} < m^3 \int_m^{2m} \frac{xdx}{x^5+1} < m^3 \int_m^{2m} \frac{dx}{x^4}$$

Thus, $\lim\limits_{m\to\infty} m^3 \cdot \int_m^{2m} \frac{x\,dx}{x^5+1} = \frac{7}{24}$

6. (a) : On simplifying, we have

$\sqrt{3}\cos A \sin C = 2 - \sin A$

$\Rightarrow \sqrt{3}\cos A \sin C + \sin A = 2$

As $\sin C \leq 1$ and maximum value of $\sqrt{3}\cos A + \sin A = 2$
So, $\sin C = 1$, *i.e.*, $C = 90°$, $A = 30°$, $B = 60°$

So, maximum area $= \frac{1}{2} \cdot 2 \cdot 2\sqrt{3} = 2\sqrt{3}$ sq. units

7. (c) : $p = abc = ra^2c \Rightarrow a^2c = \frac{p}{r}$

and $a^2 + b^2 + c^2 = a^2 + c^2 + r^2a^2$

$$= c^2 + \frac{(1+r^2)}{2}a^2 + \frac{(1+r^2)}{2}a^2$$

$$\geq 3 \cdot \sqrt[3]{c^2 \cdot \frac{(1+r^2)}{2}a^2 \cdot \frac{a^2(1+r^2)}{2}} = 3 \cdot \left(\frac{p(1+r^2)}{2r}\right)^{2/3}$$

8. (a) : Notice that A and B are on the opposite sides of the plane. So, $PA + PB$ is minimum when P is on the plane *i.e.*, it is the point of intersection of plane and line joining points A and B.

9. (b) : Use in a triangle,

$\tan A + \tan B + \tan C = \tan A \tan B \tan C$

10. (b) : If number of cubes are T_1, T_2, T_3, T_4 on top layer and B_1, B_2, B_3, B_4 on bottom layer then only restriction is no B_i can come out before corresponding T_i.

So, total possibilities $= \frac{8!}{2!2!2!2!} = 2520$

11. (b) : Simplify to get, $\log a = -a + \frac{1}{a}$

If $a > 1$ then L.H.S. > 0 and R.H.S. < 0.
So, no solution.
If $a < 1$ then L.H.S. < 0 and R.H.S. > 0. So no solution in this case also.
So, only possible value of $a = 1$.
Hence only 1 pair exists, *i.e.*, $(a, b) = (1, 1)$.

12. (a) : Let $u(x) = \left(1 + \frac{1}{x}\right)^x$ and

$$v(x) = \log\left(1 + \frac{1}{x}\right) - \frac{1}{1+x},\ x > 0$$

then $u'(x) = u(x) \cdot v(x)$ and $v'(x) = \frac{-1}{x(1+x)^2} < 0$

So, $v(x)$ is decreasing and $\lim\limits_{x\to\infty} v(x) = 0$.

Also, $u(x)$ is increasing.

$\therefore \quad f(x) = \frac{u(x)}{u\left(\frac{1}{x}\right)} = \frac{\text{increasing}}{\text{decreasing}} \to$ increasing function

and $g(x) = u(x) - u\left(\frac{1}{x}\right) = (\text{increasing}) - (\text{decreasing})$

= increasing function

13. (d) : We have, $A^2 + 3A + 2I = O$

$\Rightarrow A = (A + I)^3 + (-I)^3$

i.e., $B = A + I$ and $C = -I$

i.e., $B = \begin{bmatrix} 0 & 1 \\ 0 & -1 \end{bmatrix}$ and $C = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$

So, $Tr(B) + Tr(C) = -1 - 2 = -3$

14. (a) **15. (a)**

$\frac{dy}{dx} = \frac{(a-x)}{(x^2 - 2x + a)^{3/2}} = 0$ only at $x = a$

Exactly one stationary point.
And for graph of C_1, $a = 2$, has stationary point at $(2, \sqrt{2})$ and $x \to \infty$, $y \to 1$ and $x \to -\infty$, $y \to -1$.

For $a = 1$, $y = \frac{x}{|x-1|}$, a vertical asymptote at $x = 1$.

16. (a) **17. (a)**

$T_1(x) = \int 1 \cdot T_0(x)\,dx = x + k$ and $\int_0^1 T_1(x)dx = 0$

gives $k = \frac{-1}{2}$

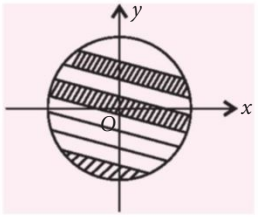
So, $T_1(x) = x - \frac{1}{2}$

Similarly, $T_2(x) = x^2 - x + \frac{1}{6}$, $T_3(x) = x^3 - \frac{3}{2}x^2 + \frac{x}{2}$

and $T_4(x) = x^2(x-1)^2 + c$

and $T_n(1) - T_n(0) = \int_0^1 \frac{dT_n(x)}{dx}dx = n \cdot \int_0^1 T_{n-1}(x)dx = 0$

18. (c) **19. (b)**



$\sin(x + y) > 0$

$\Rightarrow x + y \in (0, \pi) \cup (2\pi, 3\pi)$ etc.

and $\sin(x + y) < 0$

$\Rightarrow x + y \in (\pi, 2\pi) \cup (3\pi, 4\pi)$ etc.

Hence, $A_1 = A_2$

and $A_1 + A_2$= Full circle area $= 100\pi$

Hence, $A_1 = A_2 = 50\pi$

YOU ASK WE ANSWER

Do you have a question that you just can't get answered?

Use the vast expertise of our MTG team to get to the bottom of the question. From the serious to the silly, the controversial to the trivial, the team will tackle the questions, easy and tough. The best questions and their solutions will be printed in this column each month.

1. Evaluate the following indefinite integral:

$$\int \frac{dx}{(x+1)^5\sqrt{x^2+2x}}$$

(Priya Jain, Gujarat)

Ans. Let $I = \int \frac{dx}{(x+1)^5\sqrt{x^2+2x}} = \int \frac{dx}{(x+1)^5\sqrt{(x+1)^2-1}}$

$$= \int \frac{dx}{(x+1)^6\sqrt{1-\frac{1}{(x+1)^2}}} = \int \frac{-t^4\,dt}{\sqrt{1-t^2}} \left[\text{Putting } \frac{1}{x+1} = t\right]$$

Here, t^4 is a polynomial of degree 4.

$$\therefore \quad I = \int \frac{-t^4\,dt}{\sqrt{1-t^2}} = (At^3+Bt^2+Ct+D)\sqrt{1-t^2} + \lambda \int \frac{dt}{\sqrt{1-t^2}} \quad \ldots(i)$$

Differentiating both sides and multiplying by $\sqrt{1-t^2}$, we get

$-t^4 = (3At^2 + 2Bt + C)(1 - t^2) - t(At^3 + Bt^2 + Ct + D) + \lambda$

Comparing the coefficients, we have

$-4A = -1, 3B = 0, 3A - 2C = 0, 2B - D = 0$ and $C + \lambda = 0$

$\Rightarrow A = 1/4, B = 0, C = 3/8, D = 0$ and $\lambda = -3/8$

From (i) $I = \left(\frac{1}{4}t^3 + \frac{3}{8}t\right)\sqrt{1-t^2} - \frac{3}{8}\int \frac{dt}{\sqrt{1-t^2}}$

$= \frac{1}{8}(2t^3 + 3t)\sqrt{1-t^2} - \frac{3}{8}\sin^{-1} t + C$, where $t = \frac{1}{x+1}$.

2. Find the range of the function:

$$y = \frac{1}{2 + \sin 3x + \cos 3x}.$$

(Sidhartha, Mumbai)

Ans. We have

$$y = \frac{1}{2 + \sin 3x + \cos 3x} = \frac{1}{2 + \sqrt{2}\sin(3x + \pi/4)}$$

The value of $\sqrt{2}\sin(3x + \pi/4)$ lies between $-\sqrt{2}$ and $\sqrt{2}$ for all $x \in R$; the denominator never becomes zero. Hence, the domain is $x \in R$.

Now, $\sqrt{2}\sin(3x + \pi/4) = \frac{1}{y} - 2$

For values of x belonging to the domain ($x \in R$), we have

$-\sqrt{2} \le \sqrt{2}\sin(3x + \pi/4) \le \sqrt{2}$

i.e., $-\sqrt{2} \le \frac{1}{y} - 2 \le \sqrt{2} \Rightarrow 2 - \sqrt{2} \le \frac{1}{y} \le 2 + \sqrt{2}$

$\Rightarrow \quad \frac{1}{2+\sqrt{2}} \le y \le \frac{1}{2-\sqrt{2}}$

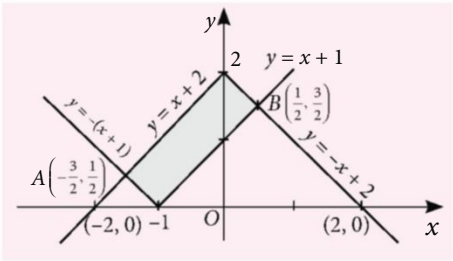
Hence, the range is $\left[\frac{1}{2+\sqrt{2}}, \frac{1}{2-\sqrt{2}}\right]$.

3. Find the area enclosed by the curves $y = |x + 1|$ and $y = 2 - |x|$. ***(Mukesh Verma, Delhi)***

Ans. We have $y = |x+1| = \begin{cases} -(x+1), & x \le -1 \\ x+1, & x > -1 \end{cases}$

and $y = -|x| + 2 = \begin{cases} x+2, & x \le 0 \\ -x+2, & x > 0 \end{cases}$

Point of intersection of $y = x + 2$ and $y = -(x + 1)$ is A *i.e.*, $\left(\frac{-3}{2}, \frac{1}{2}\right)$ and point of interaction of $y = -x + 2$ and $y = x + 1$ is B *i.e.*, $\left(\frac{1}{2}, \frac{3}{2}\right)$.



Required area = area of shaded region

$$= \int_{-3/2}^{-1}[(x+2)-(-x-1)]dx + \int_{-1}^{0}[(x+2)-(x+1)]dx + \int_{0}^{1/2}[(-x+2)-(x+1)]dx$$

$$= \int_{-3/2}^{-1}(2x+3)dx + \int_{-1}^{0}1dx + \int_{0}^{1/2}(-2x+1)dx$$

$$= \left[x^2+3x\right]_{-3/2}^{-1} + [x]_{-1}^{0} + \left[-x^2+x\right]_{0}^{1/2}$$

$$= (1-3) - \left(\frac{9}{4} - \frac{9}{2}\right) + 1 + \left(\frac{-1}{4} + \frac{1}{2}\right) = \frac{3}{2} \text{ sq.units.}$$

MPP-10 CLASS XI ANSWER KEY

1.	(b)	**2.**	(b)	**3.**	(a)	**4.**	(a)	**5.**	(d)
6.	(d)	**7.**	(a)	**8.**	(c)	**9.**	(b,c)	**10.**	(a,b)
11.	(d)	**12.**	(a,b,c)	**13.**	(c)	**14.**	(b)	**15.**	(c)
16.	(c)	**17.**	(6)	**18.**	(1)	**19.**	(2)	**20.**	(6)

MATH archives

Math Archives, as the title itself suggests, is a collection of various challenging problems related to the topics of JEE (Main & Advanced) Syllabus. This section is basically aimed at providing an extra insight and knowledge to the candidates preparing for JEE (Main & Advanced). In every issue of MT, challenging problems are offered with detailed solution. The reader's comments and suggestions regarding the problems and solutions offered are always welcome.

1. α, β, γ are the distinct roots of $x^3 = x^2 + 1$. The equation whose roots are $\alpha^2 + \beta^3 + \gamma^4, \beta^2 + \gamma^3 + \alpha^4, \gamma^2 + \alpha^3 + \beta^4$ is

(a) $y^3 - 10y^2 - 33y - 37 = 0$

(b) $y^3 - 10y^2 + 33y - 37 = 0$

(c) $y^3 + 10y^2 + 33y - 37 = 0$

(d) $y^3 - 10y^2 - 33y - 37 = 0$

2. There are 3 rows containing 2 seats in each row. The number of ways in which 3 persons can be seated such that no row remains empty is P, then $\frac{P}{16} =$

(a) 1 (b) 2

(c) 3 (d) 4

3. Tangent are drawn to the circle $x^2 + y^2 = 1$ at the points where it is meet by the circles $x^2 + y^2 - (\lambda + 6)x + (8 - 2\lambda)y - 3 = 0, \lambda$ being parameter. The locus of points of intersection of these tangents is $px + qy + 10 = 0$, where $p + q =$

(a) 1 (b) 3

(c) -1 (d) -3

4. Let α^k, where $k = 0, 1, 2, \ldots, 2013$ are the 2014^{th} roots of unity. If z_1 and z_2 be any two complex number such that $|z_1| = |z_2| = \frac{1}{\sqrt{2014}}$, then the value of $\sum_{k=0}^{2013} |z_1 + \alpha^k z_2|^2$, is equal to

(a) 4028 (b) 0

(c) 2 (d) 2014

5. Let $I_1 = \int_0^1 \frac{dx}{1+\sqrt[3]{x}}$, $I_2 = \int_0^1 \frac{dx}{1+\sqrt[4]{x}}$, then the value of $4I_1 + 3I_2$, is equal to

(a) 3 (b) 4 (c) 6 (d) 7

6. A die is rolled 5 times. The probability there are at least two equal numbers among the outcomes obtained is

(a) $\frac{319}{324}$ (b) $\frac{49}{54}$ (c) $\frac{13}{18}$ (d) $\frac{4}{9}$

7. The minimum integral value of α for which the quadratic equation $(\cot^{-1}\alpha)x^2 - (\tan^{-1}\alpha)^{\frac{3}{2}}x + 2(\cot^{-1}\alpha)^2 = 0$, has both positive roots, is

(a) 1 (b) 2 (c) 3 (d) 4

8. If $\frac{1+x+x^2}{1-x} = a_0 + a_1x + a_2x^2 + a_3x^3 + \ldots.$

and $\sum_{r=1}^{2010} a_r = p + q \times 10 + r \times 100 + s \times 1000$, where p, q, r, s are the digits of four digit number, $(p, q, r, s \in W)$ then

(a) $p = 6$ (b) $s = 9$ (c) $r = 0$ (d) $q = 7$

9. If $\cos1 + \cos2 + \cos3 + \ldots. + \cos n = S_n$, then $\lim_{n\to\infty} \frac{S_n}{n} =$

(a) 1 (b) 2

(c) does not exists (d) 0

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10. If $f(x)=f(x+1)+f(x+2)$ for all $x\in R$, $g(x)=\dfrac{x}{x-1}$ and $g(g(x))=g^2(x)$, $g(g(g(x)))=g^3(x)$, ...
$f(f(x))=f^2(x)$, $f(f(f(x)))=f^3(x)$, ... and
$f(1)=\dfrac{2}{3}$ and $f(g^{22}(3))=\dfrac{1}{3}$, then the value of $f(0)$ =

(a) $\dfrac{1}{3}$ (b) $\dfrac{2}{3}$ (c) 1 (d) 0

SOLUTIONS

1. **(b) :** $\alpha^2+\beta^3+\gamma^4=\alpha^2+\beta^3+\gamma^3+\gamma$
$=\alpha^2+\beta^2+1+\gamma^2+\gamma+1$
$=\alpha^2+\beta^2+\gamma^2+2+\gamma=3+\gamma$ and $y=3+x$
$\Rightarrow (y-3)^3=(y-3)^2+1$
$\Rightarrow y^3-10y^2+33y-37=0$

2. **(c) :** R_1, R_2, R_3 contain 1 person each.
$\therefore \quad {}^2C_1\cdot{}^2C_1\cdot{}^2C_1\cdot 3!=P$
$\Rightarrow P=48 \quad \therefore \dfrac{P}{16}=\dfrac{48}{16}=3$

3. **(a) :** $\alpha x+\beta y-1=0$ and $(\lambda+6)x+(2\lambda-8)y+2=0$ represent same type.
$\Rightarrow -2\alpha=\lambda+6$
$\Rightarrow -2\beta=2\lambda-8 \Rightarrow 2\alpha-\beta+10=0$

4. **(c) :** $\displaystyle\sum_{k=0}^{2013}|z_1+\alpha^k z_2|^2=2014(|z_1|^2+|z_2|^2)$
$=2014\left(\dfrac{2}{2014}\right)=2$

5. **(b) :** $I_1=\displaystyle\int_0^1\frac{dx}{1+\sqrt[3]{x}}=3\log 2-\frac{3}{2}$

$I_2=\displaystyle\int_0^1\frac{dx}{1+\sqrt[4]{x}}=\frac{10}{3}-4\log 2$

$\therefore \quad 4I_1+3I_2=4$

6. **(b)**

7. **(b) :** $D\ge 0 \Rightarrow \tan^{-1}\alpha\ge\dfrac{\pi}{3} \Rightarrow \alpha\ge\sqrt{3}$

Also, $\tan^{-1}\alpha>0 \Rightarrow \alpha>0$

$af(0)>0\Rightarrow 2(\cot^{-1}\alpha)^3>0 \Rightarrow a\in R$

So, minimum integral value is 2.

8. **(c) :** $(1+x+x^2)(1+x+x^2+....\infty)=a_0+a_1x+a_2x^2+....$

$a_1=2=1+1$ $a_2=1+1+1=3$
$a_3=1+1+1=3$ $a_4=1+1+1=3$

$a_{2010}=1+1+1=3 \quad \therefore \displaystyle\sum_{r=1}^{2010}a_r=2+2009(3)=6029$

9. **(d) :** $S_n=\dfrac{2\cos\left(\dfrac{n+1}{2}\right)\sin\dfrac{n}{2}}{2\sin\dfrac{1}{2}}$

$=\dfrac{\sin\left(n+\dfrac{1}{2}\right)-\sin\dfrac{1}{2}}{2\sin\dfrac{1}{2}}$

$\displaystyle\lim_{n\to\infty}\frac{S_n}{n}=0$ as S_n is a finite quantity for every n

10. **(c) :** Since $g^{22}(x)=x \Rightarrow g^{22}(3)=3$

$\therefore \quad f(g^{22}(3))=f(3)=\dfrac{1}{3}$

Now, $f(1)=f(2)+f(3) \Rightarrow f(2)=\dfrac{2}{3}-\dfrac{1}{3}=\dfrac{1}{3}$

$\therefore \quad f(0)=f(1)+f(2) \Rightarrow f(0)=\dfrac{2}{3}+\dfrac{1}{3}=1$

CLASS XII

CBSE DRILL

Practice Paper 2019

Time Allowed : 3 hours | Maximum Marks : 100

GENERAL INSTRUCTIONS

(i) All questions are compulsory.
(ii) This question paper contains **29** questions.
(iii) Questions **1-4** in Section-A are very short-answer type questions carrying **1** mark each.
(iv) Questions **5-12** in Section-B are short-answer type questions carrying **2** marks each.
(v) Questions **13-23** in Section-C are long-answer-**I** type questions carrying **4** marks each.
(vi) Questions **24-29** in Section-D are long-answer-**II** type questions carrying **6** marks each.

SECTION - A

1. If the matrix $A=\begin{bmatrix}1 & -2 & 3\\ 1 & 2 & 1\\ x & 2 & -3\end{bmatrix}$ is singular, find x.

2. Find the principal value of $\cot^{-1}\left(-\frac{1}{\sqrt{3}}\right)$.

3. If $\vec{a}=\hat{i}+2\hat{j}-\hat{k}$, $\vec{b}=2\hat{i}+\hat{j}+\hat{k}$ and $\vec{c}=5\hat{i}-4\hat{j}+3\hat{k}$, then find the value of $(\vec{a}+\vec{b})\cdot\vec{c}$.

4. Find $\int_{-1}^{1} x^3 e^{x^4}\,dx$.

SECTION - B

5. Let $A = \{x \in Z : 0 \le x \le 12\}$. Show that $R = \{(a, b) : |a - b|$ is a multiple of $4\}$ is both reflexive and symmetric.

6. If $A=\begin{bmatrix}1 & 0\\ -1 & 7\end{bmatrix}$, find k so that $A^2 = 8A + kI$.

7. Let $A=R-\left\{\frac{3}{5}\right\}$ and $B=R-\left\{\frac{7}{5}\right\}$.

Let $f: A \to B: f(x)=\frac{7x+4}{5x-3}$ and

$g: B \to A: g(y)=\frac{3y+4}{5y-7}$. Show that $(g\,o\,f) = I_A$.

8. The cartesian equation of a line is given by $6x - 2 = 3y + 1 = 2z - 2$. Write the equation in standard form and find the direction cosines of the given line.

9. A particle moves along the curve $6y = x^3 + 2$. Find the points on the curve at which y-coordinate is changing 8 times as fast as the x-coordinate.

10. If $y = e^x \log(\sin 2x)$, find $\frac{dy}{dx}$.

11. If $P(A) = 0.4, P(B) = p, P(A \cup B) = 0.6$ and A, B are given to be independent events, then find the value of p.

12. The mean and variance of a binomial distribution are 12 and 3 respectively. Find the probability distribution.

SECTION - C

13. At what points will the tangent to the curve $y = 2x^3 - 15x^2 + 36x - 21$ be parallel to x-axis. Also, find the equation of tangents at those points.

OR

Water is leaking from a conical funnel at the rate of 5 cm^3/sec. If the radius of the base of the funnel is 10 cm and its height is 20 cm, find the rate at which the water level is dropping when it is 5 cm from the top.

14. Evaluate : $\int_0^{\pi/2} \sin^4 x\,dx$

15. Form the differential equation of the family of all ellipses having foci on the x-axis and centre at the origin.

OR

Solve the differential equation $x(1 + y^2)\,dx - y(1 + x^2)\,dy = 0$, given that $y = 0$ when $x = 1$.

16. Differentiate $\cot^{-1}\left(\frac{1-x}{1+x}\right)$ w.r.t. x.

17. Solve : $\tan^{-1}\left(\frac{x-1}{x-2}\right)+\tan^{-1}\left(\frac{x+1}{x+2}\right)=\frac{\pi}{4}$.

18. A card from a pack of 52 cards is lost. From the remaining cards of the pack, two cards are drawn and are found to be both spades. Find the probability of the lost card being a spade.

OR

The probability of specific problem being solved independently by A and B are $\frac{1}{2}$ and $\frac{1}{3}$ respectively.

If both try to solve the problem independently, find the probability that

(i) the problem is solved

(ii) exactly one of them solves the problem.

19. Let $f : N \to Y$ be a function defined by $f(x) = 4x^2 + 12x + 15$ and Y = range (f). Show that f is invertible and find f^{-1}.

20. Given, $\vec{a}=\hat{i}+2\hat{j}+3\hat{k}, \vec{b}=-\hat{i}+2\hat{j}+\hat{k}$ and $\vec{c}=3\hat{i}+\hat{j}$.

Find a unit vector in the direction of resultant of these vectors. Also, find a vector $\vec{r}$ which is normal to both $\vec{a}$ and $\vec{b}$. What is the inclination of $\vec{r}$ to $\vec{c}$?

21. Find the area of the smaller region bounded by the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ and the straight line $\frac{x}{a}+\frac{y}{b}=1$.

22. Show that the operation $*$ on Z, defined by $a * b = a + b + 1$ for all $a, b \in Z$ satisfies

(i) associative law (ii) commutative law

(iii) Find the identity element in Z.

(iv) What is the inverse of an element of $a \in Z$?

23. If $x^y \cdot y^x = 1$, find $\frac{dy}{dx}$.

SECTION - D

24. If $A=\begin{bmatrix}-1 & 1 & 2\\ 1 & 2 & 3\\ 3 & 1 & 1\end{bmatrix}$, find A^{-1} by using elementary column transformations.

OR

A school wants to award its students for performance in dance, music and acting with a total cash award of ₹ 6000. Three times the award money for acting added to that given for dance amounts to ₹ 11000. The award money given for dance and acting together is double the one given for music. Represent the above situation algebraically and find the award money for each performance, using matrix method.

25. If the following function $f(x)$ is continuous at $x = 0$, find the value of a and c.

$$f(x)=\begin{cases}\frac{\sin(a+1)x+\sin x}{x}, & \text{if } x<0\\ c, & \text{if } x=0\\ \frac{\sqrt{x+bx^2}-\sqrt{x}}{bx^{3/2}}, & \text{if } x>0\end{cases}$$

OR

If $x = 3\sin t - \sin 3t$, $y = 3\cos t - \cos 3t$, find $\frac{d^2y}{dx^2}$ at $t=\frac{\pi}{3}$.

26. Find the distance of the point (1, –2, 3) from the plane $x - y + z = 5$ measured parallel to the line $\frac{x-1}{2}=\frac{y-3}{3}=\frac{z+2}{-6}$.

OR

Find the co-ordinates and length of foot of the perpendicular from the point $P(7, 14, 5)$ to the plane $2x + 4y - z = 2$. Also, find the image of the point P in the plane.

27. A manufacturer makes two products, A and B. Product A sells at ₹ 200 per unit and takes 30 minutes to make. Product B sells at ₹ 300 per unit and takes 1 hour to make. There is a permanent order of 14 units of product A and 16 units of product B. A working week consists of 40 hours of production and the weekly turnover must not be less than ₹ 10000. If the profit on each of product A is ₹ 20 and on product B is ₹ 30, then how many of each should be produced so that the profit is maximum? Also, find the maximum profit. Solve the problem graphically.

28. Show that the altitude of a right circular cone of maximum curved surface which can be inscribed in a sphere of radius r is $\frac{4r}{3}$.

29. Evaluate : $\int e^{2x}\sin(3x+1)\,dx$.

SOLUTIONS

1. For singular matrix, $|A| = 0$

$$\therefore \begin{vmatrix} 1 & -2 & 3 \\ 1 & 2 & 1 \\ x & 2 & -3 \end{vmatrix} = 0$$

$\Rightarrow 1(-6-2) + 2(-3-x) + 3(2-2x) = 0$

$\Rightarrow -8 - 8x = 0 \quad \Rightarrow \quad x = -1$

2. $\cot^{-1}\left(-\frac{1}{\sqrt{3}}\right) = \pi - \cot^{-1}\left(\frac{1}{\sqrt{3}}\right) = \pi - \cot^{-1}\left(\cot\frac{\pi}{3}\right)$

$$= \pi - \frac{\pi}{3} = \frac{2\pi}{3} \in (0, \pi)$$

3. $\vec{a} + \vec{b} = (\hat{i} + 2\hat{j} - \hat{k}) + (2\hat{i} + \hat{j} + \hat{k}) = 3\hat{i} + 3\hat{j}$

$\therefore \quad (\vec{a} + \vec{b}) \cdot \vec{c} = (3\hat{i} + 3\hat{j}) \cdot (5\hat{i} - 4\hat{j} + 3\hat{k}) = 15 - 12 + 0 = 3$

4. Let $f(x) = x^3 e^{x^4}$

Then, $f(-x) = (-x)^3 e^{(-x)^4} = -x^3 e^{x^4} = -f(x)$

Hence, $f(x)$ is an odd function.

$\therefore \quad \int_{-1}^{1} x^3 e^{x^4}\,dx = 0$

5. Clearly, $A = \{0, 1, 2, 3, 4, ..., 10, 11, 12\}$.

(i) Reflexive : Let a be an arbitrary element of A.

Then, $a - a = 0$, which is a multiple of 4.

$\therefore \quad aRa$ for all $a \in A$.

So, R is reflexive.

(ii) Symmetry : Let aRb.

Then, $aRb \Rightarrow |a - b|$ is a multiple of 4

$\Rightarrow |-(a - b)|$ is a multiple of 4

$\Rightarrow |b - a|$ is a multiple of 4 $\Rightarrow bRa$.

$\therefore \quad R$ is symmetric.

6. We have, $A = \begin{bmatrix} 1 & 0 \\ -1 & 7 \end{bmatrix}$

$$\therefore \quad A^2 = \begin{bmatrix} 1 & 0 \\ -1 & 7 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ -1 & 7 \end{bmatrix} = \begin{bmatrix} 1-0 & 0+0 \\ -1-7 & 0+49 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ -8 & 49 \end{bmatrix};$$

$$(8A + kI) = 8\begin{bmatrix} 1 & 0 \\ -1 & 7 \end{bmatrix} + k\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 8 & 0 \\ -8 & 56 \end{bmatrix} + \begin{bmatrix} k & 0 \\ 0 & k \end{bmatrix} = \begin{bmatrix} 8+k & 0 \\ -8 & 56+k \end{bmatrix}$$

$$\therefore \quad A^2 = 8A + kI \Rightarrow \begin{bmatrix} 1 & 0 \\ -8 & 49 \end{bmatrix} = \begin{bmatrix} 8+k & 0 \\ -8 & 56+k \end{bmatrix}$$

$\Rightarrow 8 + k = 1$ and $56 + k = 49 \Rightarrow k = -7$.

7. Let $x \in A$. Then,

$$(g\,o\,f)(x) = g[f(x)] = g\left(\frac{7x+4}{5x-3}\right)$$

$$= g(y), \text{ where } y = \frac{7x+4}{5x-3} \qquad \text{...(i)}$$

$$= \frac{3y+4}{5y-7} = \frac{3\left(\frac{7x+4}{5x-3}\right)+4}{5\left(\frac{7x+4}{5x-3}\right)-7} \qquad \text{[using (i)]}$$

$$= \frac{(21x+12+20x-12)}{(5x-3)} \times \frac{(5x-3)}{(35x+20-35x+21)}$$

$$= \frac{41x}{41} = x = I_A(x) \quad \therefore \quad (g\,o\,f) = I_A$$

8. The given equation may be written as

$$6\left(x - \frac{1}{3}\right) = 3\left(y + \frac{1}{3}\right) = 2(z-1)$$

$$\Rightarrow \frac{\left(x - \frac{1}{3}\right)}{\left(\frac{1}{6}\right)} = \frac{\left(y + \frac{1}{3}\right)}{\left(\frac{1}{3}\right)} = \frac{(z-1)}{\left(\frac{1}{2}\right)}$$

$$\Rightarrow \frac{\left(x - \frac{1}{3}\right)}{1} = \frac{\left(y + \frac{1}{3}\right)}{2} = \frac{(z-1)}{3} \text{ [on dividing each by 6],}$$

which is the standard form of the given equation in cartesian form. The direction ratios of the given line are 1, 2, 3.

Also, $\sqrt{1^2+2^2+3^2}=\sqrt{14}$

So, the direction cosines of the given line are

$\frac{1}{\sqrt{14}}, \frac{2}{\sqrt{14}}, \frac{3}{\sqrt{14}}$.

9. The given curve is $6y = x^3 + 2$ ... (i)

Differentiating (i) w.r.t. t, we get $6\frac{dy}{dt} = 3x^2\frac{dx}{dt}$ ...(ii)

But $\frac{dy}{dt} = 8\frac{dx}{dt}$ (given),

$\therefore$ From (ii), $6\times 8\frac{dx}{dt} = 3x^2\frac{dx}{dt} \Rightarrow 48 = 3x^2$

$\Rightarrow x^2 = 16 \Rightarrow x = 4, -4$

From (i), when $x = 4$, $6y = 4^3 + 2 = 66 \Rightarrow y = 11$,

when $x = -4$, $6y = (-4)^3 + 2 = -62 \Rightarrow y = -\frac{31}{3}$

Hence, the required points on the curve are (4, 11), $\left(-4, -\frac{31}{3}\right)$.

10. We have, $y = e^x \log(\sin 2x)$ ...(i)

Differentiating (i) w.r.t. x, we get

$$\frac{dy}{dx} = e^x\cdot\frac{d}{x}\{\log(\sin 2x)\} + \log(\sin 2x)\cdot\frac{d}{dx}(e^x)$$

$$= e^x\cdot\left\{\frac{1}{\sin 2x}\cdot\cos 2x\cdot 2\right\} + \log(\sin 2x)\cdot e^x$$

$= 2e^x \cot 2x + e^x \log(\sin 2x) = e^x[2\cot 2x + \log(\sin 2x)]$.

11. Since A and B are independent events

$\therefore P(A \cap B) = P(A)\cdot P(B)$

We have, $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$\Rightarrow P(A \cup B) = P(A) + P(B) - P(A)\cdot P(B)$

$\Rightarrow 0.6 = 0.4 + p - 0.4p$

$\Rightarrow 0.6p = 0.2 \quad \therefore p = \frac{1}{3}$.

12. Here, $np = 12$ and $npq = 3$

$\Rightarrow 12q = 3 \Rightarrow q = \frac{1}{4}$

$\therefore p = 1 - q = \frac{3}{4}$ and $n\cdot\frac{3}{4} = 12 \Rightarrow n = 16$

$\therefore$ The Binomial distribution is $(q+p)^n$ *i.e.*, $\left(\frac{1}{4}+\frac{3}{4}\right)^{16}$

13. Given curve is $y = 2x^3 - 15x^2 + 36x - 21$ ...(i)

Differentiating (i) w.r.t. x, we get

$\frac{dy}{dx} = 2\cdot 3x^2 - 15\cdot 2x + 36\cdot 1 = 6x^2 - 30x + 36$.

Let $P(x_1, y_1)$ be a point on the given curve at which the tangent is parallel to x-axis *i.e.*, has slope 0.

$\therefore$ Slope of tangent at $P = \left(\frac{dy}{dx}\right)_P = 6x_1^2 - 30x_1 + 36 = 0$

$\Rightarrow x_1^2 - 5x_1 + 6 = 0 \Rightarrow (x_1 - 2)(x_1 - 3) = 0 \Rightarrow x_1 = 2, 3$

As $P(x_1, y_1)$ lies on the given curve,

$y_1 = 2x_1^3 - 15x_1^2 + 36x_1 - 21$

When $x_1 = 2$, $y_1 = 2\cdot 2^3 - 15\cdot 2^2 + 36\times 2 - 21 = 7$;

when $x_1 = 3$, $y_1 = 2\cdot 3^3 - 15\cdot 3^2 + 36\times 3 - 21 = 6$

Thus, there are two points (2, 7) and (3, 6) on the given curve where the tangents are parallel to x-axis.

The equation of tangent at (2, 7) is $y - 7 = 0\ (x - 2)$

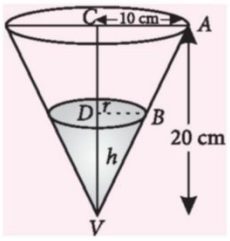
$\Rightarrow y - 7 = 0$.

The equation of tangent at (3, 6) is $y - 6 = 0\ (x - 3)$

$\Rightarrow y - 6 = 0$.

OR

Let r and h be the radius and height at any time t of the cone respectively.



Since Δs DVB and CVA are similar,

$\therefore \frac{DV}{CV} = \frac{DB}{CA} \Rightarrow \frac{h}{20} = \frac{r}{10}$

$\Rightarrow r = \frac{h}{2}$

$\therefore$ Volume of water at any time $(V) = \frac{1}{3}\pi r^2 h$

$= \frac{1}{3}\pi\cdot\left(\frac{h}{2}\right)^2\cdot h = \frac{\pi}{12}h^3$ ...(i)

Diff. (i) w.r.t. t, we get $\frac{dV}{dt} = \frac{\pi}{12}\cdot 3h^2\frac{dh}{dt} = \frac{\pi}{4}h^2\frac{dh}{dt}$

Since water is leaking from the conical funnel at the rate of 5 cm^3/sec,

$\therefore \frac{dV}{dt} = -5 \quad \left(\frac{dV}{dt}\text{ is } -\text{ve, because } V \text{ is decreasing}\right)$

$\Rightarrow -5 = \frac{\pi}{4}h^2\frac{dh}{dt} \Rightarrow \frac{dh}{dt} = -\frac{20}{\pi h^2}$

When the water level is 5 cm from the top, $h = 20 - 5 = 15$ cm,

$\left(\frac{dh}{dt}\right)_{(h=15)} = -\frac{20}{\pi\times(15)^2} = -\frac{4}{45\pi}$ cm/sec.

Hence, the rate at which the water level is dropping

$= \frac{4}{45\pi}$ cm/sec.

14. $\int_0^{\pi/2}\sin^4 x\,dx = \frac{1}{4}\int_0^{\pi/2}(2\sin^2 x)^2\,dx$

$$= \frac{1}{4}\cdot\int_0^{\pi/2}(1-\cos 2x)^2\,dx = \frac{1}{4}\int_0^{\pi/2}(1-2\cos 2x+\cos^2 2x)dx$$

$$= \frac{1}{4}\cdot\int_0^{\pi/2}\left[1-2\cos 2x+\frac{(1+\cos 4x)}{2}\right]dx$$

$$= \frac{1}{4}\cdot\int_0^{\pi/2}\left(\frac{3}{2}-2\cos 2x+\frac{1}{2}\cos 4x\right)dx$$

$$= \frac{3}{8}\cdot\int_0^{\pi/2}dx-\frac{1}{2}\int_0^{\pi/2}\cos 2x\,dx+\frac{1}{8}\int_0^{\pi/2}\cos 4x\,dx$$

$$= \frac{3}{8}\cdot[x]_0^{\pi/2}-\frac{1}{2}\cdot\left[\frac{\sin 2x}{2}\right]_0^{\pi/2}+\frac{1}{8}\cdot\left[\frac{\sin 4x}{4}\right]_0^{\pi/2}$$

$$= \frac{3}{8}\left(\frac{\pi}{2}-0\right)-\frac{1}{4}(\sin\pi-0)+\frac{1}{32}(\sin 2\pi-0) = \frac{3\pi}{16}$$

15. The general equation of an ellipse having foci on x-axis and centre at the origin is given by

$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ ... (i), where a and b are the parameters.

Differentiating (i) w.r.t. x, we get

$\frac{2x}{a^2}+\frac{2y}{b^2}\cdot\frac{dy}{dx}=0 \Rightarrow \frac{yy_1}{b^2}=\frac{-x}{a^2}$, where $\frac{dy}{dx}=y_1$

$\Rightarrow \frac{yy_1}{x}=\frac{-b^2}{a^2}$ ...(ii)

Differentiating (ii) w.r.t. x, we get

$$\frac{x\cdot\frac{d}{dx}(yy_1)-yy_1\cdot\frac{d}{dx}(x)}{x^2}=0$$

$\Rightarrow x[yy_2+(y_1)^2]-yy_1=0$

$\Rightarrow (xy)y_2+x(y_1)^2-yy_1=0$

Hence, $(xy)\frac{d^2y}{dx^2}+x\left(\frac{dy}{dx}\right)^2-y\left(\frac{dy}{dx}\right)=0$ is the required differential equation.

OR

The given differential equation is

$x(1+y^2)\,dx - y(1+x^2)dy = 0$

$\Rightarrow x(1+y^2)\,dx = y(1+x^2)\,dy$

$\Rightarrow \frac{y}{(1+y^2)}dy=\frac{x}{(1+x^2)}dx$

$\Rightarrow \int\frac{y}{(1+y^2)}dy=\int\frac{x}{(1+x^2)}dx$

$\Rightarrow \frac{1}{2}\int\frac{2y}{(1+y^2)}dy=\frac{1}{2}\int\frac{2x}{(1+x^2)}dx$

$\Rightarrow \frac{1}{2}\log(1+y^2)=\frac{1}{2}\log(1+x^2)+\frac{1}{2}\log|C_1|$,

where C_1 is an arbitrary constant

$\Rightarrow \log(1+y^2)=\log(1+x^2)+\log|C_1|$

$\Rightarrow \frac{(1+y^2)}{(1+x^2)}=\pm C_1=C$ (say)

$\Rightarrow (1+y^2)=C(1+x^2)$ ...(i)

Putting $x = 1$ and $y = 0$ in (i), we get $C=\frac{1}{2}$.

Putting $C=\frac{1}{2}$ in eqn. (i), we get $x^2 - 2y^2 = 1$.

Hence, $x^2 - 2y^2 = 1$ is the required solution.

16. Let $y=\cot^{-1}\left(\frac{1-x}{1+x}\right)$ ...(i)

Putting $x = \tan\theta$ in (i), we get

$$y=\cot^{-1}\left(\frac{1-\tan\theta}{1+\tan\theta}\right)=\cot^{-1}\left\{\tan\left(\frac{\pi}{4}-\theta\right)\right\}$$

$$=\cot^{-1}\left[\cot\left\{\frac{\pi}{2}-\left(\frac{\pi}{4}-\theta\right)\right\}\right]=\cot^{-1}\left[\cot\left(\frac{\pi}{4}+\theta\right)\right]$$

$$=\frac{\pi}{4}+\theta=\frac{\pi}{4}+\tan^{-1}x \quad [\because x=\tan\theta\Rightarrow\theta=\tan^{-1}x]$$

$\therefore\ y=\frac{\pi}{4}+\tan^{-1}x \Rightarrow \frac{dy}{dx}=\frac{d}{dx}\left\{\frac{\pi}{4}+\tan^{-1}x\right\}=\frac{1}{(1+x^2)}$.

17. We have, $\tan^{-1}\left(\frac{x-1}{x-2}\right)+\tan^{-1}\left(\frac{x+1}{x+2}\right)=\frac{\pi}{4}$

$$\Rightarrow \tan^{-1}\left[\frac{\frac{(x-1)}{(x-2)}+\frac{(x+1)}{(x+2)}}{1-\frac{(x-1)}{(x-2)}\cdot\frac{(x+1)}{(x+2)}}\right]=\frac{\pi}{4}$$

$$\Rightarrow \tan^{-1}\left\{\frac{(x-1)(x+2)+(x+1)(x-2)}{(x^2-4)-(x^2-1)}\right\}=\frac{\pi}{4}$$

$$\Rightarrow \tan^{-1}\left\{\frac{(x^2+x-2)+(x^2-x-2)}{-3}\right\}=\frac{\pi}{4}$$

$$\Rightarrow \frac{2x^2-4}{-3}=\tan\frac{\pi}{4}=1\Rightarrow 2x^2-4=-3$$

$$\Rightarrow 2x^2=1\Rightarrow x^2=\frac{1}{2}\Rightarrow x=\pm\frac{1}{\sqrt{2}}.$$

18. Let E_1, E_2, E_3 and E_4 be the events of losing a card of spades, clubs, hearts and diamonds respectively.

Let E be the event of drawing 2 spades cards from the remaining 51 cards.

Now, $P(E_1) = P(E_2) = P(E_3) = P(E_4) = \frac{13}{52}=\frac{1}{4}$

$P(E/E_1)$ = Probability of drawing 2 spade cards given that a card of spade is missing

$$= \frac{^{12}C_2}{^{51}C_2} = \frac{12\times11}{51\times50} = \frac{22}{425}$$

$$P(E/E_2) = P(E/E_3) = P(E/E_4) = \frac{^{13}C_2}{^{51}C_2} = \frac{13\times12}{51\times50} = \frac{26}{425}$$

$\therefore$ By Bayes' theorem, $P\left(\frac{E_1}{E}\right) = \frac{P(E_1)\cdot P(E/E_1)}{\sum_{i=1}^{4} P(E_i)\cdot P(E/E_i)}$

$$= \frac{\frac{1}{4}\times\frac{22}{425}}{\frac{1}{4}\times\frac{22}{425}+\frac{1}{4}\times\frac{26}{425}+\frac{1}{4}\times\frac{26}{425}+\frac{1}{4}\times\frac{26}{425}}$$

$$= \frac{22}{22+3\times26} = \frac{22}{100} = 0.22$$

OR

Let E_1 and E_2 be the events that A and B solves the problem respectively.

Then, $P(E_1) = \frac{1}{2}$ and $P(E_2) = \frac{1}{3}$

$\Rightarrow P(\bar{E}_1) = \left(1-\frac{1}{2}\right) = \frac{1}{2}$ and $P(\bar{E}_2) = \left(1-\frac{1}{3}\right) = \frac{2}{3}$

Clearly, E_1 and E_2 are independent events.

$\therefore\ P(E_1 \cap E_2) = P(E_1)\times P(E_2) = \left(\frac{1}{2}\times\frac{1}{3}\right) = \frac{1}{6}$

(i) P(the problem is solved)
$= P$(at least one of A and B solves the problem)
$= P(E_1$ or $E_2) = P(E_1 \cup E_2)$
$= P(E_1) + P(E_2) - P(E_1 \cap E_2)$
$= \left(\frac{1}{2}+\frac{1}{3}-\frac{1}{6}\right) = \frac{4}{6} = \frac{2}{3}.$

(ii) P(exactly one of them solves the problem)
$= P[(E_1$ and not $E_2)$ or $(E_2$ and not $E_1)]$
$= P(E_1 \cap \bar{E}_2) + P(E_2 \cap \bar{E}_1)$
$= P(E_1)\times P(\bar{E}_2) + P(E_2)\times P(\bar{E}_1)$
$= \left(\frac{1}{2}\times\frac{2}{3}\right)+\left(\frac{1}{3}\times\frac{1}{2}\right) = \left(\frac{1}{3}+\frac{1}{6}\right) = \frac{3}{6} = \frac{1}{2}.$

19. Let $x_1, x_2 \in N$, then $f(x_1) = f(x_2)$
$\Rightarrow 4x_1^2 + 12x_1 + 15 = 4x_2^2 + 12x_2 + 15$
$\Rightarrow\ 4(x_1^2 - x_2^2) + 12(x_1 - x_2) = 0$
$\Rightarrow\ (x_1^2 - x_2^2) + 3(x_1 - x_2) = 0 \Rightarrow\ (x_1 - x_2)(x_1 + x_2 + 3) = 0$
$\Rightarrow\ x_1 - x_2 = 0 \quad [\because x_1 + x_2 + 3 \neq 0]$
$\Rightarrow\ x_1 = x_2$
$\therefore\ f$ is one-one
Also, range $(f) = Y$. So, f is onto.
Thus, f is one-one onto and therefore invertible.
Let $y \in Y$. Since, f is onto, there exists $x \in N$ such that $y = f(x)$.
Now, $y = f(x)$
$\Rightarrow\ y = 4x^2 + 12x + 15 \Rightarrow\ y = (2x+3)^2 + 6$
$\Rightarrow (2x+3) = \sqrt{y-6} \Rightarrow\ x = \frac{1}{2}(\sqrt{y-6} - 3)$
$\Rightarrow\ f^{-1}(y) = \frac{1}{2}(\sqrt{y-6} - 3)$
$\therefore\ f^{-1} : Y \to N$ is defined by $f^{-1}(y) = \frac{1}{2}(\sqrt{y-6} - 3).$

20. The resultant of given vectors is given by
$(\vec{a} + \vec{b} + \vec{c}) = 3\hat{i} + 5\hat{j} + 4\hat{k}$
$\therefore\ |\vec{a} + \vec{b} + \vec{c}| = \sqrt{3^2 + 5^2 + 4^2} = 5\sqrt{2}$
$\therefore$ Unit vector in the direction of the resultant vector
$= \frac{1}{5\sqrt{2}}(3\hat{i} + 5\hat{j} + 4\hat{k})$
Let $\vec{r} = \vec{a} \times \vec{b}$, then

$$\vec{r} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & 3 \\ -1 & 2 & 1 \end{vmatrix} = -4\hat{i} - 4\hat{j} + 4\hat{k}$$

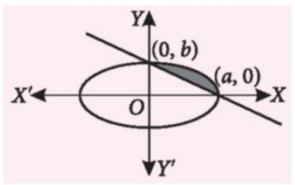
$\therefore\ |\vec{r}| = \sqrt{(-4)^2 + (-4)^2 + 4^2} = 4\sqrt{3}$
Let θ be the angle between $\vec{r}$ and $\vec{c}$, then

$$\theta = \cos^{-1}\left\{\frac{\vec{r}\cdot\vec{c}}{|\vec{r}||\vec{c}|}\right\} = \cos^{-1}\left\{\frac{(-4\hat{i} - 4\hat{j} + 4\hat{k})\cdot(3\hat{i} + \hat{j})}{(4\sqrt{3})\cdot(\sqrt{10})}\right\}$$

$$\Rightarrow\ \theta = \cos^{-1}\left(\frac{-4}{\sqrt{30}}\right)$$

$\therefore$ Angle between $\vec{r}$ and $\vec{c} = \pi - \cos^{-1}\left(\frac{4}{\sqrt{30}}\right)$

21. We have, $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ (ellipse) and $\frac{x}{a} + \frac{y}{b} = 1$ (line).



$\therefore$ Required area = Area of shaded region

$$= \int_0^a (y \text{ of the ellipse})dx - \int_0^a (y \text{ of the line})dx$$

$$= \int_0^a \frac{b}{a}\cdot\sqrt{a^2 - x^2}\,dx - \int_0^a \frac{b(a-x)}{a}dx$$

$$= \frac{b}{a}\cdot\left[\frac{x}{2}\sqrt{a^2 - x^2} + \frac{a^2}{2}\sin^{-1}\frac{x}{a}\right]_0^a - \frac{b}{a}\cdot\left[ax - \frac{x^2}{2}\right]_0^a$$

$$= \frac{ab}{2}(\sin^{-1}1 - \sin^{-1}0) - \frac{b}{a}\left[a^2 - \frac{a^2}{2}\right]$$

$$= \left(\frac{\pi ab}{4} - \frac{ab}{2}\right) \text{sq units.}$$

22. Given, $a * b = a + b + 1$, $a \in Z$, $b \in Z$.

(i) Associative : For all $a, b, c \in Z$, we have

$(a * b) * c = (a + b + 1) * c = (a + b + 1) + c + 1$

$= a + b + c + 2$

Now, $a * (b * c) = a * (b + c + 1) = a + (b + c + 1) + 1$

$= a + b + c + 2$

$\therefore \quad (a * b) * c = a * (b * c)$.

(ii) Commutative : For all $a, b \in Z$, we have

$a * b = a + b + 1 = b + a + 1$ [$\because a + b = b + a \ \forall \ a, b \in Z$]

$= b * a$

(iii) Existence of identity element

Let e be the identity element in Z.

Then, $a * e = a \Rightarrow a + e + 1 = a$

$\Rightarrow e = -1$.

Thus, $-1 \in Z$ is the identity element for $*$.

(iv) Existence of Inverse

Let $a \in Z$ and let its inverse be b. Then,

$a * b = -1 \Rightarrow a + b + 1 = -1 \Rightarrow b = -(2 + a)$

Clearly, $2 \in Z$, $a \in Z \Rightarrow -(2 + a) \in Z$

Thus, each $a \in Z$ has $-(2 + a) \in Z$ as its inverse.

23. Given $x^y \cdot y^x = 1$

Taking log on both sides, we get

$\Rightarrow \ (y \log x) + (x \log y) = 0$ ...(i) [$\because \log 1 = 0$]

On differentiating both sides of (i) w.r.t. x, we get

$$y \cdot \frac{d}{dx}(\log x) + (\log x) \cdot \frac{d}{dx}(y) + x \cdot \frac{d}{dx}(\log y) + (\log y) \cdot \frac{d}{dx}(x) = 0$$

$$\Rightarrow \ y \cdot \frac{1}{x} + (\log x) \cdot \frac{dy}{dx} + x \cdot \frac{1}{y} \cdot \frac{dy}{dx} + (\log y) \cdot 1 = 0$$

$$\Rightarrow \left(\log x + \frac{x}{y}\right) \cdot \frac{dy}{dx} = -\left(\log y + \frac{y}{x}\right)$$

$$\Rightarrow \frac{(y \log x + x)}{y} \cdot \frac{dy}{dx} = \frac{-(x \log y + y)}{x}$$

$$\Rightarrow \frac{dy}{dx} = \frac{-y(x \log y + y)}{x(y \log x + x)}$$

24. Given $A = \begin{bmatrix} -1 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{bmatrix}$, then $A = AI_3$

$$\Rightarrow \begin{bmatrix} -1 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{bmatrix} = A \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Applying $C_1 \leftrightarrow C_2$

$$\Rightarrow \begin{bmatrix} 1 & -1 & 2 \\ 2 & 1 & 3 \\ 1 & 3 & 1 \end{bmatrix} = A \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Apply $C_2 \to C_2 + C_1$, $C_3 \to C_3 - 2C_1$

$$\Rightarrow \begin{bmatrix} 1 & 0 & 0 \\ 2 & 3 & -1 \\ 1 & 4 & -1 \end{bmatrix} = A \begin{bmatrix} 0 & 1 & 0 \\ 1 & 1 & -2 \\ 0 & 0 & 1 \end{bmatrix}$$

Apply $C_1 \to C_1 + 2C_3$, $C_2 \to C_2 + 2C_3$

$$\Rightarrow \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & -1 \\ -1 & 2 & -1 \end{bmatrix} = A \begin{bmatrix} 0 & 1 & 0 \\ -3 & -3 & -2 \\ 2 & 2 & 1 \end{bmatrix}$$

Apply $C_3 \to C_3 + C_2$

$$\Rightarrow \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -1 & 2 & 1 \end{bmatrix} = A \begin{bmatrix} 0 & 1 & 1 \\ -3 & -3 & -5 \\ 2 & 2 & 3 \end{bmatrix}$$

Apply $C_1 \to C_1 + C_3$, $C_2 \to C_2 - 2C_3$

$$\Rightarrow \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = A \begin{bmatrix} 1 & -1 & 1 \\ -8 & 7 & -5 \\ 5 & -4 & 3 \end{bmatrix}$$

$$\Rightarrow \ A^{-1} = \begin{bmatrix} 1 & -1 & 1 \\ -8 & 7 & -5 \\ 5 & -4 & 3 \end{bmatrix}.$$

OR

Let the amount of award for performance in dance, music and acting be ₹x, ₹y and ₹z respectively.

Then, $x + y + z = 6000$ ...(i)

$x + 0y + 3z = 11000$ ...(ii)

and $x + z = 2y \Rightarrow x - 2y + z = 0$ ...(iii)

The above system of equations can be written in matrix form as $AX = B$,

where $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 0 & 3 \\ 1 & -2 & 1 \end{bmatrix}$, $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$ and $B = \begin{bmatrix} 6000 \\ 11000 \\ 0 \end{bmatrix}$.

Now, $|A| = \begin{vmatrix} 1 & 1 & 1 \\ 1 & 0 & 3 \\ 1 & -2 & 1 \end{vmatrix} = \begin{vmatrix} 1 & 0 & 0 \\ 1 & -1 & 2 \\ 1 & -3 & 0 \end{vmatrix}$

[Applying $C_2 \to C_2 - C_1$ and $C_3 \to C_3 - C_1$]

$$= 1 \cdot \begin{vmatrix} -1 & 2 \\ -3 & 0 \end{vmatrix} = 6 \neq 0$$

$\therefore$ A is non singular and the above system has a unique solution given by $X = A^{-1}B$.

Let A_{ij} be the cofactors of elements a_{ij} in A.

Now, the cofactors of the elements of $|A|$ are given by

$$A_{11} = \begin{vmatrix} 0 & 3 \\ -2 & 1 \end{vmatrix} = 6,\ A_{12} = -\begin{vmatrix} 1 & 3 \\ 1 & 1 \end{vmatrix} = 2,\ A_{13} = \begin{vmatrix} 1 & 0 \\ 1 & -2 \end{vmatrix} = -2;$$

$$A_{21} = -\begin{vmatrix} 1 & 1 \\ -2 & 1 \end{vmatrix} = -3,\ A_{22} = \begin{vmatrix} 1 & 1 \\ 1 & 1 \end{vmatrix} = 0,\ A_{23} = -\begin{vmatrix} 1 & 1 \\ 1 & -2 \end{vmatrix} = 3;$$

$$A_{31} = \begin{vmatrix} 1 & 1 \\ 0 & 3 \end{vmatrix} = 3,\ A_{32} = -\begin{vmatrix} 1 & 1 \\ 1 & 3 \end{vmatrix} = -2,\ A_{33} = \begin{vmatrix} 1 & 1 \\ 1 & 0 \end{vmatrix} = -1.$$

$$\therefore\ (\text{adj } A) = \begin{bmatrix} 6 & 2 & -2 \\ -3 & 0 & 3 \\ 3 & -2 & -1 \end{bmatrix}' = \begin{bmatrix} 6 & -3 & 3 \\ 2 & 0 & -2 \\ -2 & 3 & -1 \end{bmatrix}$$

$$\text{Now, } A^{-1} = \frac{1}{|A|} \cdot (\text{adj } A) = \frac{1}{6} \cdot \begin{bmatrix} 6 & -3 & 3 \\ 2 & 0 & -2 \\ -2 & 3 & -1 \end{bmatrix}$$

$$\therefore\ X = A^{-1}B = \frac{1}{6} \cdot \begin{bmatrix} 6 & -3 & 3 \\ 2 & 0 & -2 \\ -2 & 3 & -1 \end{bmatrix} \begin{bmatrix} 6000 \\ 11000 \\ 0 \end{bmatrix}$$

$$\Rightarrow X = \frac{1}{6} \cdot \begin{bmatrix} 36000 - 33000 + 0 \\ 12000 + 0 + 0 \\ -12000 + 33000 - 0 \end{bmatrix} = \frac{1}{6} \begin{bmatrix} 3000 \\ 12000 \\ 21000 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 500 \\ 2000 \\ 3500 \end{bmatrix}$$

Hence, the award money for dance, music and acting is ₹ 500, ₹ 2000 and ₹ 3500 respectively.

25. We have, $f(0) = c$.

$$\lim_{x \to 0^-} f(x) = \lim_{h \to 0} f(0 - h)$$

$$= \lim_{h \to 0} \frac{\sin(a+1)(-h) + \sin(-h)}{(-h)} = \lim_{h \to 0} \frac{-\{\sin(a+1)h + \sin h\}}{-h}$$

$$= \lim_{h \to 0} \frac{\sin(a+1)h + \sin h}{h} = \lim_{h \to 0} \frac{2\sin\left(\frac{a}{2}+1\right)h \cdot \cos\frac{ah}{2}}{h}$$

$$= 2 \cdot \lim_{h \to 0} \left\{ \frac{\sin\left(\frac{a}{2}+1\right)h}{\left(\frac{a}{2}+1\right)h} \cdot \left(\frac{a}{2}+1\right) \cdot \cos\frac{ah}{2} \right\}$$

$$= 2\left(\frac{a}{2}+1\right) \cdot \lim_{h \to 0} \frac{\sin\left(\frac{a}{2}+1\right)h}{\left(\frac{a}{2}+1\right)h} \cdot \lim_{h \to 0} \cos\frac{ah}{2} = (a+2).$$

$$\lim_{x \to 0^+} f(x) = \lim_{h \to 0} f(0 + h)$$

$$= \lim_{h \to 0} \left\{ \frac{[\sqrt{h + bh^2} - \sqrt{h}]}{bh^{3/2}} \times \frac{[\sqrt{h + bh^2} + \sqrt{h}]}{[\sqrt{h + bh^2} + \sqrt{h}]} \right\}$$

$$= \lim_{h \to 0} \frac{(h + bh^2 - h)}{bh^{3/2}(\sqrt{h + bh^2} + \sqrt{h})} = \lim_{h \to 0} \frac{bh^2}{bh^2(\sqrt{1 + bh} + 1)}$$

$$= \lim_{h \to 0} \frac{1}{(\sqrt{1 + bh} + 1)} = \frac{1}{2}.$$

Since $f(x)$ is continuous at $x = 0$, we have

$$f(0) = \lim_{x \to 0^-} f(x) = \lim_{x \to 0^+} f(x) \Rightarrow c = \frac{1}{2} \text{ and } a + 2 = \frac{1}{2}$$

$$\therefore\ c = \frac{1}{2} \text{ and } a = \frac{-3}{2}.$$

OR

We have, $x = 3 \sin t - \sin 3t$

$$\Rightarrow \frac{dx}{dt} = 3\cos t - 3\cos 3t \qquad \text{...(i)}$$

and $y = 3 \cos t - \cos 3t$

$$\Rightarrow \frac{dy}{dt} = -3\sin t + 3\sin 3t \qquad \text{... (ii)}$$

$$\therefore\ \frac{dy}{dx} = \frac{(dy/dt)}{(dx/dt)} = \frac{-3\sin t + 3\sin 3t}{3\cos t - 3\cos 3t} = \frac{\sin 3t - \sin t}{\cos t - \cos 3t}$$

$$= \frac{2\cos 2t \sin t}{2\sin 2t \sin t} = \cot 2t$$

$$\therefore \frac{d^2y}{dx^2} = -2\text{cosec}^2 2t \cdot \frac{dt}{dx} = \frac{-2\text{cosec}^2 2t}{(dx/dt)} = \frac{-2\text{cosec}^2 2t}{3(\cos t - \cos 3t)}$$

$$\therefore \left(\frac{d^2y}{dx^2}\right)_{\left(t = \frac{\pi}{3}\right)} = \frac{-2\,\text{cosec}^2\left(\frac{2\pi}{3}\right)}{3\left(\cos\frac{\pi}{3} - \cos\pi\right)} = -2 \times \left(\frac{2}{\sqrt{3}}\right)^2 \times \frac{1}{3\left(\frac{1}{2}+1\right)}$$

$$= \left(-2 \times \frac{4}{3} \times \frac{2}{9}\right) = -\frac{16}{27}.$$

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26. The equation of the given plane is

$x - y + z = 5$ ...(i)

The equation of the given line is

$$\frac{x-1}{2} = \frac{y-3}{3} = \frac{z+2}{-6}.$$ ...(ii)

The equation of the line passing through the point $P(1, -2, 3)$ and parallel to line (ii) is given by

$$\frac{x-1}{2} = \frac{y+2}{3} = \frac{z-3}{-6} = \lambda \text{ (say)}.$$ ...(iii)

Any point say Q, on line (iii) is $(2\lambda + 1, 3\lambda - 2, -6\lambda + 3)$.

For some value of λ, let the point $Q(2\lambda + 1, 3\lambda - 2, -6\lambda + 3)$ lie on plane (i).

$\therefore \quad (2\lambda + 1) - (3\lambda - 2) + (-6\lambda + 3) = 5$

$\Rightarrow -7\lambda = -1 \Rightarrow \lambda = \frac{1}{7}$.

So, the coordinates of Q are $\left(\frac{9}{7}, \frac{-11}{7}, \frac{15}{7}\right)$.

$$\text{Distance } PQ = \sqrt{\left(\frac{9}{7}-1\right)^2 + \left(\frac{-11}{7}+2\right)^2 + \left(\frac{15}{7}-3\right)^2}$$

$$= \sqrt{\left(\frac{2}{7}\right)^2 + \left(\frac{3}{7}\right)^2 + \left(\frac{-6}{7}\right)^2} = \frac{1}{7}\sqrt{4+9+36}$$

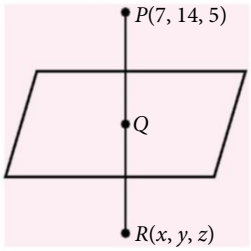
$$= \frac{1}{7} \times \sqrt{49} = \left(\frac{1}{7} \times 7\right) = 1 \text{ unit.}$$

OR

The given plane is $2x + 4y - z = 2$ ...(i)

The d.r.'s of the normal to this plane are 2, 4, –1.

The equation of the line passing through the point $P(7, 14, 5)$ and the perpendicular to the given plane (i) is given by



$$\frac{x-7}{2} = \frac{y-14}{4} = \frac{z-5}{-1} = \lambda \text{ (say)}$$

Any point say Q on this line is $(2\lambda + 7, 4\lambda + 14, -\lambda + 5)$.

For some value of λ, let the point $Q(2\lambda + 7, 4\lambda + 14, -\lambda + 5)$ lie on the plane (i).

$\therefore \quad 2(2\lambda + 7) + 4(4\lambda + 14) - (-\lambda + 5) = 2$

$\Rightarrow \quad 21\lambda = -63 \Rightarrow \lambda = -3.$

$\therefore$ The coordinates of Q are $[2 \times (-3) + 7, 4 \times (-3) + 14, -(-3) + 5]$ *i.e.*, $Q(1, 2, 8)$.

$$\therefore \quad PQ = \sqrt{(7-1)^2 + (14-2)^2 + (5-8)^2}$$

$$= \sqrt{6^2 + (12)^2 + (-3)^2} = \sqrt{36+144+9}$$

$$= \sqrt{189} = 3\sqrt{21} \text{ units.}$$

Let $R(x, y, z)$ be the image of P in the plane (i).

Then, Q is the midpoint of PR.

$$\therefore \quad \frac{7+x}{2} = 1, \quad \frac{14+y}{2} = 2 \text{ and } \frac{5+z}{2} = 8$$

$\Rightarrow \quad x = -5, y = -10, z = 11.$

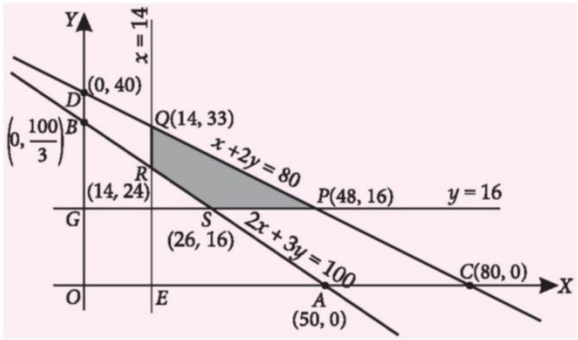
Hence, the image of $P(7, 14, 5)$ in the given plane is $R(-5, -10, 11)$.

27. Let x be the number of units of the product A and y be the number of units of the product B produced. Let z be the total profit. According to question x and y must satisfy the following conditions :

$200x + 300y \geq 10000$ or $2x + 3y \geq 100$

$$\frac{1}{2}x + y \leq 40 \text{ or } x + 2y \leq 80$$

$x \geq 14, y \geq 16, x \geq 0, y \geq 0$



Mathematical formulation of the LPP will be

Maximize $z = 20x + 30y$

subject to constraints :

$2x + 3y \geq 100$

$x + 2y \leq 80$

$x \geq 14, y \geq 16$

$x \geq 0, y \geq 0$

On plotting, we get the corner points of the feasible region as $S(26, 16)$, $P(48, 16)$, $Q(14, 33)$ and $R(14, 24)$

Maximize $z = 20x + 30y$

$\therefore$ At $S(26, 16)$, $z = 1000$

At $P(48, 16)$, $z = 1440$

At $Q(14, 33)$, $z = 1270$

At $R(14, 24)$, $z = 1000$

Hence, the maximum profit is ₹ 1440 and it is attained when 48 units of product A and 16 units of product B are produced.

28. Let x be the radius of the inscribed right circular cone and h be its height, then

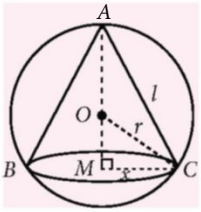
$OM^2 + MC^2 = OC^2$ (Pythagoras theorem)

$\Rightarrow (h - r)^2 + x^2 = r^2$ ($\because$ $OA = r$)

$\Rightarrow x^2 = 2hr - h^2$ ...(i)

Let S is the curved surface area of the cone, then

$$S = \pi . xl = \pi x\sqrt{h^2 + x^2}$$



$$\Rightarrow S^2 = \pi^2 x^2 (h^2 + x^2)$$
$$= \pi^2 (2hr - h^2)(h^2 + 2hr - h^2)$$
$$= \pi^2 (2hr - h^2) 2hr$$
$$= 2\pi^2 r (2h^2 r - h^3).$$

Since $S > 0$, therefore, S is maximum iff S^2 is maximum, so we need to find the value of h for which S^2 is maximum. Let $S^2 = f(h)$

$\therefore$ $f(h) = 2\pi^2 r(2h^2 r - h^3)$, $0 < h < 2r$ ...(i)

Differentiating (i) w.r.t. h, we get

$f'(h) = 2\pi^2 r(4hr - 3h^2)$ and $f''(h) = 2\pi^2 r(4r - 6h)$

Now, $f'(h) = 0 \Rightarrow 2\pi^2 r(4hr - 3h^2) = 0$

$\Rightarrow h(4r - 3h) = 0$ but $0 < h < 2r \Rightarrow h = \frac{4}{3}r$.

Also $f''\left(\frac{4}{3}r\right) = 2\pi^2 r\left(4r - 6.\frac{4}{3}r\right) = -8\pi^2 r^2 < 0$

$\Rightarrow f(h)$ is maximum when $h = \frac{4}{3}r$

$\Rightarrow S^2$ is maximum when $h = \frac{4}{3}r$

$\Rightarrow$ S is maximum when $h = \frac{4}{3}r$

Therefore, the curved surface area of the inscribed cone is maximum when its altitude = $\frac{4}{3}r$.

29. Let $I = \int e^{2x} \sin(3x+1)dx$

$$= \sin(3x+1)\cdot\frac{e^{2x}}{2} - \int \cos(3x+1)\cdot 3\cdot\frac{e^{2x}}{2}dx$$

$$= \frac{1}{2}e^{2x}\sin(3x+1) - \frac{3}{2}\int \cos(3x+1)\cdot e^{2x}dx + C_1$$

$$= \frac{1}{2}e^{2x}\sin(3x+1) - \frac{3}{2}\left[\cos(3x+1)\cdot\frac{e^{2x}}{2} - \int -\sin(3x+1)\cdot 3\cdot\frac{e^{2x}}{2}dx\right] + C_2$$

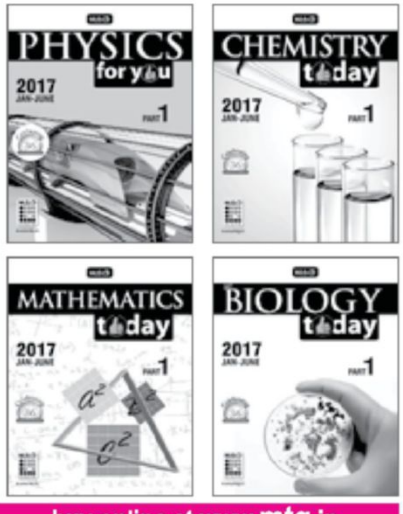
$$= \frac{1}{2}e^{2x}\sin(3x+1) - \frac{3}{4}e^{2x}\cos(3x+1) - \frac{9}{4}I + C_2$$

$$\left(1+\frac{9}{4}\right)I = \frac{1}{4}e^{2x}[2\sin(3x+1) - 3\cos(3x+1)] + C_2$$

$$\Rightarrow \frac{13}{4}\int e^{2x}\sin(3x+1)dx = \frac{1}{4}e^{2x}[2\sin(3x+1) - 3\cos(3x+1)] + C_2$$

$$\Rightarrow \int e^{2x}\sin(3x+1)dx = \frac{1}{13}e^{2x}[2\sin(3x+1) - 3\cos(3x+1)] + C$$

AVAILABLE BOUND VOLUMES



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MPP-10 MONTHLY Practice Problems

Class XII

This specially designed column enables students to self analyse their extent of understanding of specified chapters. Give yourself four marks for correct answer and deduct one mark for wrong answer.
Self check table given at the end will help you to check your readiness.

Probability

Total Marks : 80 **Time Taken : 60 Min.**

Only One Option Correct Type

1. A signal which can be green or red with probability $\frac{4}{5}$ and $\frac{1}{5}$ respectively, is received by station A and then transmitted to station B. The probability of each station receiving the signal correctly is $\frac{3}{4}$. If the signal received at station B is green then the probability that the original signal was green is

(a) $\frac{3}{5}$ (b) $\frac{6}{7}$ (c) $\frac{20}{23}$ (d) $\frac{9}{20}$

2. It is given that mean and variance of a binomial variate X are 2 and 1 respectively. The probability that X takes a value greater or equal to 1 is

(a) $\frac{13}{16}$ (b) $\frac{5}{16}$ (c) $\frac{15}{16}$ (d) $\frac{1}{8}$

3. If A and B are two independent events such that $P(A' \cap B) = 2/15$ and $P(A \cap B') = 1/6$, then $P(B)$ is

(a) 1/5 (b) 1/6 (c) 2/3 (d) 5/6.

4. A man takes a step forward with probability 0.4 and backward with probability 0.6. The probability that at the end of 11 steps he is one step away from the starting point is

(a) $420(0.24)^5$ (b) $420(0.24)^6$
(c) $462(0.24)^5$ (d) $462(0.24)^6$

5. A bag contains a white and b black balls. Two players, A and B alternatively draw a ball from the bag, replacing the ball each time after the draw till one of them draws a white ball and wins the game. A begins the game. If the probability of A winning the game is three times that of B, the ratio $a : b$ is

(a) 1 : 1 (b) 1 : 2
(c) 2 : 1 (d) none of these

6. The sum and product of the mean and variance of a binomial distribution are 24 and 128 respectively. The distribution is

(a) $\left(\frac{1}{7}+\frac{1}{8}\right)^{12}$ (b) $\left(\frac{1}{4}+\frac{3}{4}\right)^{16}$

(c) $\left(\frac{1}{6}+\frac{5}{6}\right)^{24}$ (d) $\left(\frac{1}{2}+\frac{1}{2}\right)^{32}$

One or More Than One Option(s) Correct Type

7. If X follows a binomial distribution with parameters $n = 8$ and $p = \frac{1}{2}$, then $P(|X - 4| \le 2)$ is

(a) $\frac{118}{128}$ (b) $\frac{119}{128}$ (c) $\frac{117}{128}$ (d) $\frac{121}{128}$

8. If A and B are two events such that $P(A) = 1/2$ and $P(B) = 2/3$, then

(a) $P(A \cup B) \ge 2/3$ (b) $P(A \cap B') \le 1/3$
(c) $1/6 \le P(A \cap B) \le 1/2$ (d) $1/6 \le P(A' \cap B) \le 1/2$.

9. An electric component manufactured by 'RASU Electronics' is tested for its defectiveness by a sophisticated testing device. Let A denote the event "the device is defective" and B the event "the testing device reveals the component to be defective". Suppose $P(A) = \alpha$ and $P(B/A) = P(B'/A') = 1 - \alpha$, where $0 < \alpha < 1$, then

(a) $P(B) = 2\alpha(1 - \alpha)$ (b) $P(A'/B) = 1/2$
(c) $P(B') = (1 - \alpha)^2 + \alpha^2$ (d) $P(A'/B') = [\alpha/(1 - \alpha)]^2$

10. Let E and F be two independent events. The probability that the exactly one of them occurs is 11/25 and the probability of none of them occurring is 2/25. If $P(T)$ denotes the probability of occurrence of the event T, then

(a) $P(E) = \frac{4}{5}, P(F) = \frac{3}{5}$ (b) $P(E) = \frac{1}{5}, P(F) = \frac{2}{5}$

(c) $P(E) = \frac{2}{5}, P(F) = \frac{1}{5}$ (d) $P(E) = \frac{3}{5}, P(F) = \frac{4}{5}$

11. Let $0 < P(A) < 1$, $0 < P(B) < 1$ and $P(A \cup B) = P(A) + P(B) - P(A)\,P(B)$, then

(a) $P(B/A) = P(B) - P(A)$

(b) $P(A' - B') = P(A') - P(B')$

(c) $P(A \cup B)' = P(A')\,P(B')$

(d) $P(A/B) = P(A)$

12. For two events A and B, if $P(A) = P(A/B) = 1/4$ and $P(B/A) = 1/2$, then

(a) A and B are independent

(b) A and B are mutually exclusive

(c) $P(A'/B) = 3/4$ (d) $P(B'/A') = 1/2$

13. A ship is fitted with three engines E_1, E_2 and E_3. The engine function independently of each other with respective probabilities $\frac{1}{2}, \frac{1}{4}$ and $\frac{1}{4}$. For the ship to be operational at least two of its engines must function. Let X denote the event that the ship is operational and let X_1, X_2 and X_3 denote respectively the events that the engines E_1, E_2 and E_3 are functioning then which of the following is true?

(a) $P[X_1^c/X] = 3/16$

(b) P [Exactly two engines of the ship are functioning/X] $= 7/8$

(c) $P[X/X_2] = 5/16$ (d) $P[X/X_1] = 5/16$

Comprehension Type

A JEE aspirant estimates that she will be successful with an 80% chance if she studies 10 hours per day, with a 60% chance if she studies 7 hours per day and with a 40% chance if she studies 4 hours per day. She further believes that she will study 10 hours, 7 hours and 4 hours per day with probabilities 0.1, 0.2 and 0.7, respectively.

14. The probability that she will be successful, is

(a) 0.28 (b) 0.38 (c) 0.48 (d) 0.58

15. Given that she does not achieve success, the probability she studied for 4 hours, is

(a) $\frac{18}{26}$ (b) $\frac{19}{26}$ (c) $\frac{20}{26}$ (d) $\frac{21}{26}$

Matrix Match Type

16. A bag contains some white and some black balls, all combinations being equally likely. The total number of balls in the bag is 12. Four balls are drawn at random from the bag without replacement.

Column-I		Column-II	
P.	Probability that all the four balls are black is equal to	1.	14/33
Q.	If the bag contains 10 black and two white balls, then the probability that all four balls are black is equal to	2.	1/5
R.	If all the four balls are black, then the probability that the bag contains 10 black balls is equal to	3.	70/429
S.	Probability that two balls are black and two are white is	4.	168/715

	P	Q	R	S		P	Q	R	S
(a)	3	1	2	4	(b)	2	1	3	2
(c)	4	1	2	3	(d)	2	3	1	4

Numerical Answer Type

17. One ticket is selected at random from 100 tickets numbered 00, 01, 02, ... , 98, 99. If X and Y denote the sum and the product of the digits on the tickets, The value of $57\,P(X = 9 \mid Y = 0)$ is

18. The least number of times a fair coin must be tossed so that the probability of getting atleast one head is atleast 0.8, is

19. A student appears for tests I, II and III. The student is successful if he passes either in test I and II or test I and III. The probabilities of the student passing in tests I, II, III are p, q, $\frac{1}{2}$ respectively. If the probability that the student is successful is $\frac{1}{2}$, then $p =$

20. If X is a binomial variate with parameters n and p, where $0 < p < 1$ such that $\frac{P(X=r)}{P(X=n-r)}$ is independent of n and r, then $2p$ equals

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SELF CHECK

No. of questions attempted .....
No. of questions correct .....
Marks scored in percentage .....

Check your score! If your score is

> 90%	EXCELLENT WORK !	You are well prepared to take the challenge of final exam.
90-75%	GOOD WORK !	You can score good in the final exam.
74-60%	SATISFACTORY !	You need to score more next time.
< 60%	NOT SATISFACTORY!	Revise thoroughly and strengthen your concepts.

MPP-10 | MONTHLY Practice Problems

Class XI

This specially designed column enables students to self analyse their extent of understanding of specified chapters. Give yourself four marks for correct answer and deduct one mark for wrong answer.
Self check table given at the end will help you to check your readiness.

Statistics and Probability

Total Marks : 80 **Time Taken : 60 Min.**

Only One Option Correct Type

1. In a class of 10 students, probability of exactly i students passing an examination is directly proportional to i^2, then the probability that exactly five students pass an examination is
(a) 1/11 (b) 5/77 (c) 25/77 (d) 10/77

2. The first of two samples has 100 items with mean 15 and S.D. 3. If the whole group has 250 items with mean 15.6 and S.D. = $\sqrt{13.44}$, then the S.D. of the second group is
(a) 5 (b) 4 (c) 6 (d) 3.52

3. The odds against the player A to win are 5 : 2 and odds in favour of another player to win are 6 : 5. If the two events are independent, find the probability that at least one player will win.
(a) 52/77 (b) 51/77 (c) 53/77 (d) 59/77

4. The mean of five observations is 4.4 and the variance is 8.24. Three of the five observations are 1, 2 and 6. The remaining two are
(a) 9, 4 (b) 7, 6 (c) 6, 5 (d) 10, 3

5. If $\bar{x}$ denotes the mean of n observations $x_1, x_2, \ldots, x_n$, then mean of the observation $x_i + 2i$, where $i = 1, 2, 3, \ldots, n$ is
(a) $\bar{x} + 2$ (b) $\bar{x} + n$
(c) $\bar{x} + 2n$ (d) $\bar{x} + n + 1$

6. The probability that in a family of five members, exactly two members have birthday on Sunday is
(a) $(12 \times 5^3) / 7^5$ (b) $(10 \times 6^2) / 7^5$
(c) 2/5 (d) $(10 \times 6^3) / 7^5$

One or More Than One Option(s) Correct Type

7. The variance of first 50 even natural numbers is
(a) 833 (b) 437 (c) $\frac{437}{4}$ (d) $\frac{833}{4}$

8. A natural number is chosen at random from the first 100 natural numbers. The probability that $x + \frac{100}{x} > 50$ is
(a) 1/10 (b) 11/50
(c) 11/20 (d) none of these

9. Mean of the numbers 1, 2, 3, ..., n with respective weights $1^2 + 1, 2^2 + 2, 3^2 + 3, \ldots, n^2 + n$ is
(a) $\frac{3n(n+1)}{2(2n+1)}$ (b) $\frac{3n^2+7n+2}{2(2n+4)}$
(c) $\frac{3n+1}{4}$ (d) $\frac{3n+1}{2}$

10. The probability that a 50 yr old man will be alive at 60 is 0.83 and the probability that a 45 yr old woman will be alive at 55 is 0.87. Then,
(a) the probability that both will be alive after 10 year is 0.7221

EXAM ALERT 2019

Exam	Date
JEE Main II	6th to 20th April
VITEEE	10th to 21st April
SRMJEEE	15th to 25th April
KEAM	22nd & 23rd April
Karnataka CET	23rd & 24th April
AMU (Engg.)	28th April
COMEDK (Engg.)	12th May
BITSAT	16th to 26th May
JEE Advanced	19th May
WB JEE	26th May

(b) atleast one of them will alive after 10 year is 0.9779
(c) atleast one of them will alive after 10 year is 0.8230
(d) the probability that both will be alive after 10 year is 0.6320

11. Let r be the range of n ($\forall n \geq 1$) observations $x_1, x_2, \ldots, x_n$. If $S = \sqrt{\dfrac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{n-1}}$, then

(a) $S < r\sqrt{\dfrac{n^2+1}{n-1}}$ (b) $S \geq r\sqrt{\dfrac{n}{n-1}}$

(c) $S = r\sqrt{\dfrac{n}{n-1}}$ (d) $S < r\sqrt{\dfrac{n}{n-1}}$

12. Three numbers are chosen at random without replacement from {1, 2, ..., 15}. Let E_1 be the event that minimum of the chosen numbers is 5 and E_2 be the event that their maximum is 10, then

(a) $P(E_1) = \dfrac{9}{91}$ (b) $P(E_2) = \dfrac{36}{455}$

(c) $P(E_1 \cap E_2) = \dfrac{4}{455}$ (d) $P(E_1 \cap E_2) = \dfrac{1}{9}$

13. All the students of a class performed poorly in Mathematics. The teacher decided to give grace marks of 10 to each of the students. Which of the following statistical measures will not change even after the grace marks were given?

(a) Median (b) Mode
(c) Variance (d) Mean

Comprehension Type

Box 1 contains three tickets bearing numbers 1, 2, 3; box 2 contains five tickets bearing numbers 3, 4, 5, 6, 7; and box 3 contains seven tickets bearing numbers 3, 4, 5, 6, 7, 8, 9. A ticket is drawn from each of the boxes. Let x_i be the number on the tickets drawn from the i^{th} box, i = 1, 2, 3.

14. The probability that $x_1 + x_2 + x_3$ is odd, is

(a) $\dfrac{29}{105}$ (b) $\dfrac{53}{105}$ (c) $\dfrac{57}{105}$ (d) $\dfrac{1}{2}$

15. The probability that x_1, x_2, x_3 are in an arithmetic progression, is

(a) $\dfrac{9}{105}$ (b) $\dfrac{10}{105}$ (c) $\dfrac{11}{105}$ (d) $\dfrac{7}{105}$

Matrix Match Type

16. Three distinct numbers a, b, c are chosen at random from the numbers 1, 2, ..., 100. The probability that

Column-I		Column-II	
P.	a, b, c are in A.P.	1.	$\dfrac{53}{161700}$
Q.	a, b, c are in G.P.	2.	1/66
R.	$\dfrac{1}{a}, \dfrac{1}{b}, \dfrac{1}{c}$ are in G.P.	3.	1/22
S.	$a + b + c$ is divisible by 2	4.	1/2

	P	Q	R	S
(a)	2	4	1	3
(b)	3	2	2	1
(c)	2	1	3	4
(d)	2	1	1	3

Numerical Answer Type

17. The papers of 4 students can be checked by any one of the 7 teachers. If the probability that all the 4 papers are checked by exactly 2 teachers is A, then the value of 49 A must be

18. Three persons A, B and C are to speak at a function along with 7 other persons. If the probability that A speaks before B and B speaks before C is $\dfrac{K}{6}$ then K is

19. The digits 1, 2, 3, 4, 5, 6, 7, 8 and 9 are written in random order to form a nine digit number. Let p be the probability that this number is divisible by 36, find 9p.

20. If the variate takes the values 0, 2, 4, 8, ..., 2^n with frequencies ${}^nC_0, {}^nC_1, \ldots, {}^nC_n$ respectively and if the mean is $\dfrac{91 \times 8}{2^n}$, then n equals

Keys are published in this issue. Search now! ☺

SELF CHECK

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No. of questions correct .....
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< 60%	NOT SATISFACTORY!	Revise thoroughly and strengthen your concepts.

9 TIPS TO SCORE HIGH IN CBSE CLASS 12th

Mathematics can be a nightmare for some students whereas fun for others. It requires a great knowledge of application of formulas and logical approach. It is one of the most important subjects for those students who look to pursue their careers in Engineering field, Accountancy, etc. Since Board exams are nearing, we are providing here 9 tips to score high in CBSE Class 12th Maths.

1. Make short notes on Theorems and Formulae: As we all know that Maths is all about theorems, formulae and concepts. So, it is always good to keep them handy so that you can read and brush them up before your practice test or exam time.
2. Solve Problem by Yourself: It is good to go through different types of problems by yourself. Make the best use of NCERT textbooks and learn each concept given in it by self-study. Solve all the exercise problems given at the end of each topic for at least 3 to 4 times over the course of time. It will help you to understand the theories and concepts more clearly.
3. Understand the Syllabus: Understanding the syllabus is very important as it helps to decide the amount of time that you need to spend on each section or topic to plan a proper study-schedule. This means if any topic or section carries less marks then spend less time on its preparation. Know the weightage of each and every topic from MTG CBSE Champion class 12 and plan your study accordingly.
4. Solve Practice Papers: It is always advisable to solve respective questions of a topic while studying it. It will help you to gain more confidence and perfection in solving Maths problems. For this, you should refer to the most recommended Mathematics books which will give you an ample number of practice papers along with easy recapitulation of concepts. You should also solve each and every problem given in the NCERT textbook and Exemplar Problems as many of these problems directly or indirectly are asked in the Board exam.
5. Clear Out Your Doubts: While solving any question if you come across any doubt and unable to solve it then it is necessary to clear it out at the first place before jumping to the next question. Leaving the doubts uncleared over a period of time may trouble you in solving them later.
6. Practice a lot: You cannot become a master in Maths in a day. You have to practice a lot because without practice you cannot do anything in Maths. As it is also said that "Practice makes a man perfect". So, keep practicing and for this, solve numerous sample papers and Previous Year's papers. This will improve your Maths problem solving skills and also help in managing time during the actual Board exam.
7. Focus on Steps: Maths is the only subject in which each step carries some marks. So, do not leave any question unattempted as you will get marks for each step even if your final answer is incorrect.
8. Draw Graphs and diagrams: Draw neat and clean graphs and diagrams wherever required. This can fetch you full marks in that particular question. For this, you need to do a good practice of graphs and diagrams in order to draw them precisely during exam.
9. Solve the known problems first: During the examination, first finish all those questions which are more familiar to you. If you come across any difficulty while solving any question then do not waste much time solving it repeatedly. You should leave that question and proceed to the next one. This will save your time to complete the rest of the paper within the stipulated time frame.

We hope the given tips will prove helpful to you in preparing for CBSE Class 12th Maths exam paper. Try these tips with dedication and practice as many Maths problems as you can to improve your accuracy and speed in solving them during the exam.

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MATHS MUSING

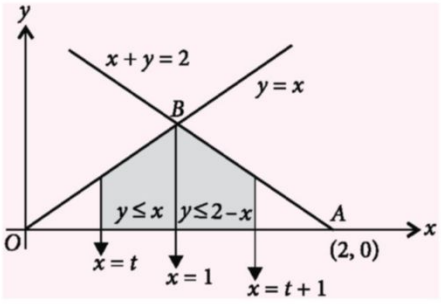
SOLUTION SET-193

1. (d) : Let us define the events in the following way.
E_1 : there will be rain, E_2 : there will not be rain,
A : the picnic will be good
Given that $P(E_1) = 0.8$, $P(E_2) = 0.2$ and $P(A|E_1) = 0.3$ and $P(A|E_2) = 0.9$
Now, by law of total probability,
$P(A) = P(A \cap E_1) + P(A \cap E_2)$
$= P(E_1) \cdot P(A|E_1) + P(E_2) \cdot P(A|E_2)$
$= 0.8 \times 0.3 + 0.2 \times 0.9 = 0.42$

2. (d) : $(a-2)(x-[x])^2 + 2(x-[x]) + a^2 = 0$ ... (i)
Now, $f(y) = (a-2)y^2 + 2y + a^2 = 0$, where $y = x - [x]$ ... (ii)
Since x cannot be an integer
when $2 < x < 3 \Rightarrow [x] = 2$ and $0 < y < 1$
$\therefore$ Equation (i) has exactly one solution in (2, 3)
$\therefore$ Equation (ii) has exactly one solution in (0, 1)
$\therefore$ $f(0) \cdot f(1) < 0$
$\Rightarrow a^2(a-2+2+a^2) < 0 \Rightarrow a^3(a+1) < 0 \Rightarrow a \in (-1, 0)$

3. (a) : $\because f$ is continuous function $\therefore \lim_{x \to 1} f(x) = f(1)$.
Put $x = 1$ in the given equation we have
$(f(1))^2 = 2f(1)$, so $f(1) = 0$ or 2.
Since $f(1) > 0$, so $f(1) = 2$.

4. (a) : Here $M = \{(x, y) : y \le x, y \ge 0 \text{ and } y \le 2 - x\}$
$N = \{(x, y) : t \le x \le t + 1 \text{ and } 0 \le t \le 1\}$
if $x \le t + 1$, $0 \le t \le 1 \Rightarrow$ Max value of $x = 2$



Area of $M \cap N = \int_t^1 x\,dx + \int_1^{t+1} (2-x)\,dx = -t^2 + t + \frac{1}{2}$

5. (a) : $|r_1 - r_2| < C_1C_2 < r_1 + r_2$
$\Rightarrow |r-3| < 5 < r+3 \Rightarrow 5 < r+3 \Rightarrow 2 < r$ ...(i)
Also, $|r-3| < 5 \Rightarrow -5 < r-3 < 5 \Rightarrow -2 < r < 8$ ...(ii)
From (i) & (ii) we get, $2 < r < 8$

6. (a, b, c, d) : $AA^T = A^TA = I$
Also $A^T = A$, so $A^2 = I \Rightarrow A$ is involutory matrix.
$\Rightarrow |A^2| = |A|^2 = 1$ or $|A| = \pm 1$.

But $|A| = \begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix} = (a+b+c)\begin{vmatrix} 1 & b & c \\ 1 & c & a \\ 1 & a & b \end{vmatrix}$
$\Rightarrow |A| = (a + b + c)(ab + bc + ca - a^2 - b^2 - c^2)$
$\Rightarrow |A| = 3abc - a^3 - b^3 - c^3$
or $= -\frac{1}{2}[(a-b)^2 + (b-c)^2 + (c-a)^2] < 0$
$\therefore$ $|A| = -1 \Rightarrow a^3 + b^3 + c^3 - 3abc = 1$

7. (b) **8. (a)**

$\frac{t_n}{a_2} = \frac{t'_n}{a_1} = 4 \Rightarrow \frac{t_n}{t'_n} = \frac{a_2}{a_1}$
$t_n = a_1 + (n-1)d_1 = 4a_2$ ... (i)
$t'_n = a_2 + (n-1)d_2 = 4a_1$ ... (ii)

Also, $S_n = 2S'_n$ *i.e.*, $\frac{n}{2}[a_1 + t_n] = 2 \cdot \frac{n}{2}[a_2 + t'_n]$
$\Rightarrow a_1 + 4a_2 = 2[a_2 + 4a_1] \Rightarrow 7a_1 = 2a_2 \Rightarrow \frac{a_1}{a_2} = \frac{2}{7}$
$\therefore$ $1 + (n-1)\frac{d_1}{a_1} = 4\frac{a_2}{a_1} = 4.\frac{7}{2} = 14$ (from (i))
$1 + (n-1)\frac{d_2}{a_2} = 4\frac{a_1}{a_2} = 4 \cdot \frac{2}{4} = \frac{8}{7}$ (from (ii))

$(n-1)\frac{d_1}{a_1} = 14 - 1 = 13$ ...(iii)
$(n-1)\frac{d_2}{a_2} = \frac{8}{7} - 1 = \frac{1}{7}$ ...(iv)

Dividing (iii) by (iv), we get
$\Rightarrow \frac{d_1}{d_2} \cdot \frac{a_2}{a_1} = 13 \cdot 7 \Rightarrow \frac{d_1}{d_2} = 13 \cdot 7 \cdot \frac{2}{7} = 26$

9. (1) : $iz^3 + z^2 + i^2z + i = 0 \Rightarrow (iz + 1)(z^2 + i) = 0$
$\Rightarrow |z| = \left|\frac{-1}{i}\right|$ or $|z^2| = -i$
$\therefore$ $|z| = 1$ in any case $\therefore$ $[|z|] = 1$

10. (c) : P → 2, Q → 3, R → 1

(P) $\frac{4(1-\cos 2x)}{2} + \frac{3(1+\cos 2x)}{2} + \sqrt{1+\sin x}$
$= \frac{7 - \cos 2x}{2} + \sqrt{1+\sin x}$
$= 3 + \sin^2 x + \sqrt{1+\sin x} = 4 + \sqrt{2}$ (Maximum)

(Q) $\cos 2\theta + \cos\theta = \frac{-9}{8} + 2\left(\cos\theta + \frac{1}{4}\right)^2$
$\Rightarrow$ Minimum $= \frac{-9}{8}$

(R) $-10 \le 3\sin x + 4\cos x - 5 \le 0$
$\Rightarrow \max|3\sin x + 4\cos x - 5| = 10$

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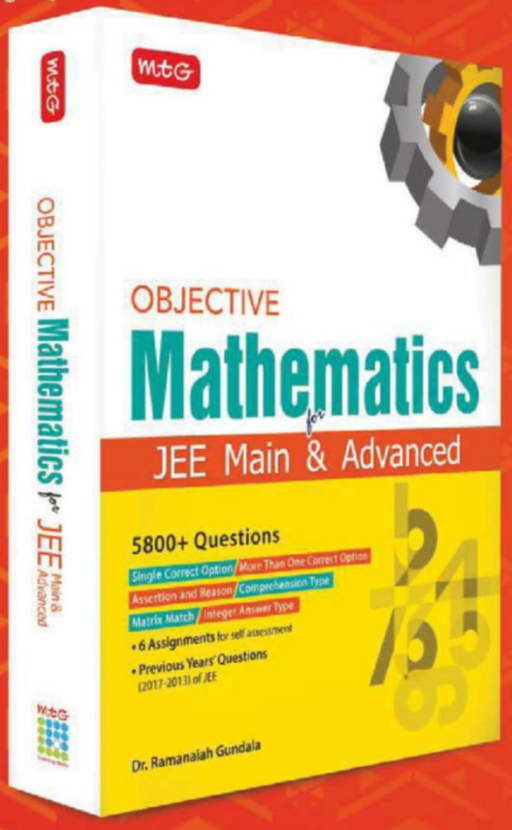
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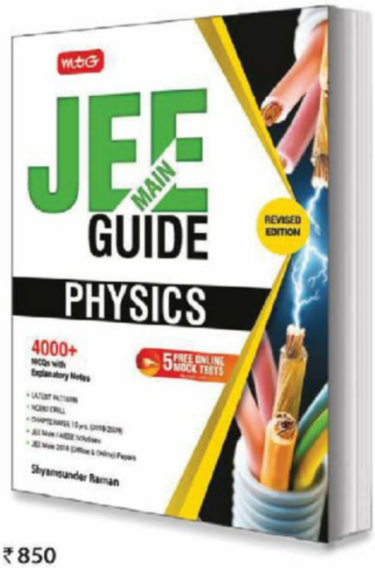

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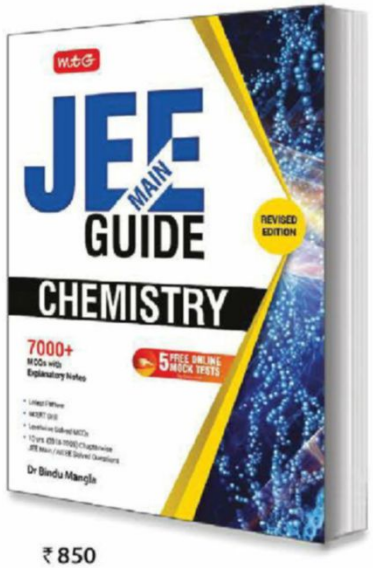

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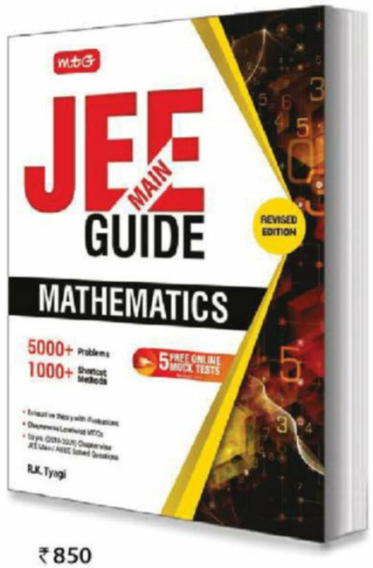

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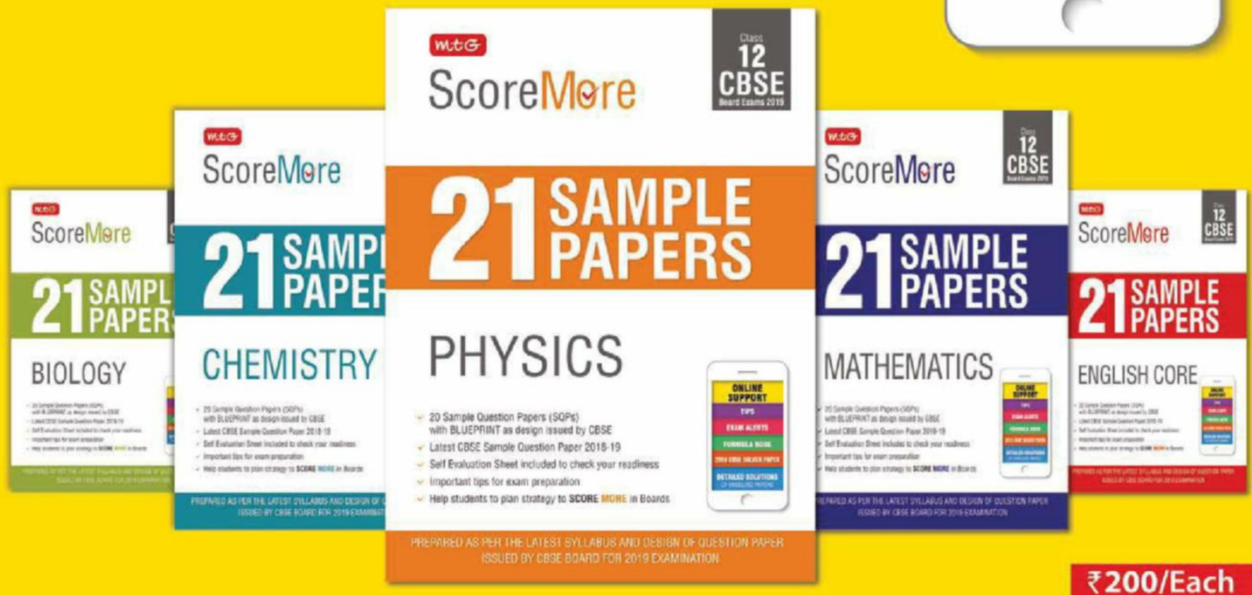
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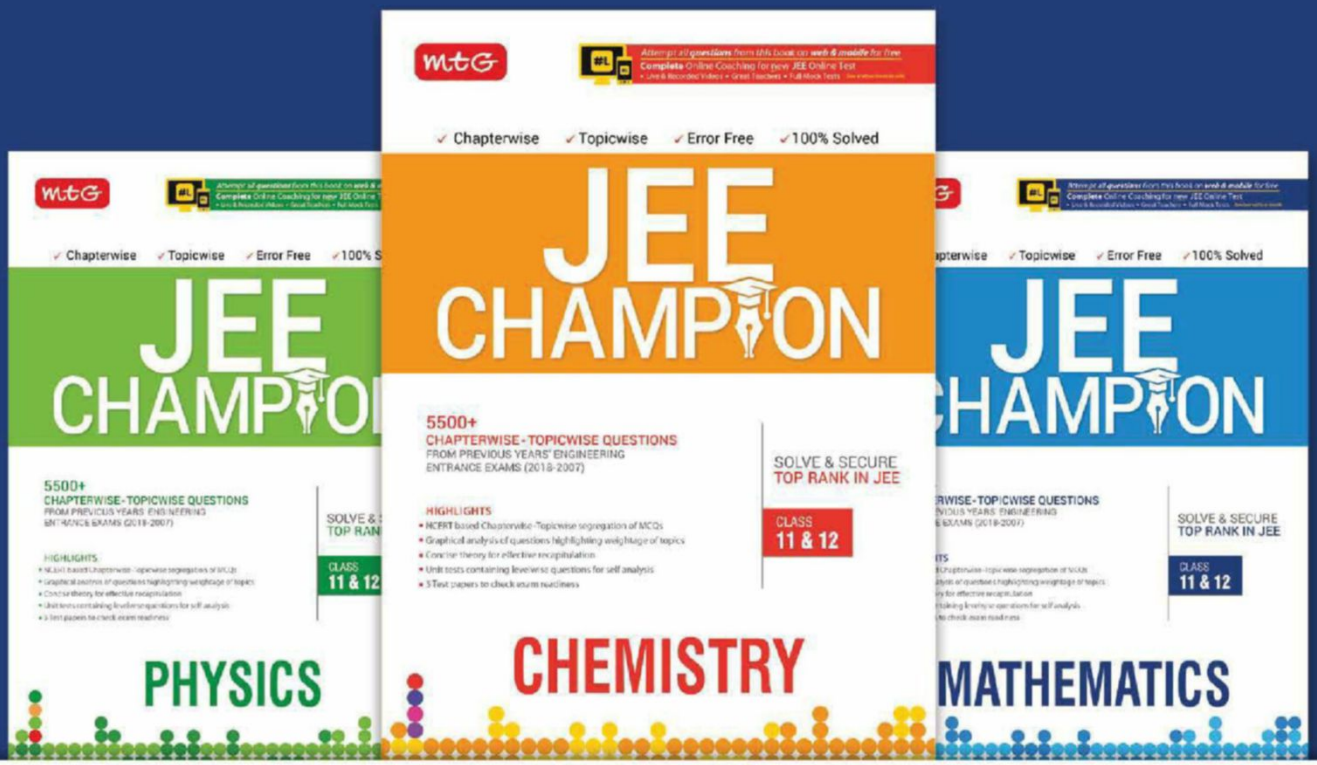

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